St Christophers the Hall - Maths Written Calculation Policy - 2023 (Created - March 2023 and reviewed January 2025)

To be reviewed: January 2027

This policy supports the White Rose Maths scheme used throughout the school. Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum. This calculation policy should be used to support children to develop a deep understanding of number and calculation. This policy has been designed to teach children through the use of concrete, pictorial and abstract (CPA) representations.

- **Concrete representation**— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.
- **Pictorial representation** – a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.
- **Abstract representation**—a pupil is now capable of representing problems by using mathematical notation, for example 12 x 2 = 24, column method of addition or subtraction.

It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations. The progressional use of **correct vocabulary is key** so children can build upon their prior learning and embed their understanding towards their Varied Fluency and Reasoning (application) Problem Solving approaches with the aim of achieving mastery.

Mathematics Mastery

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

How to use the policy:

This mathematics policy is a guide for all staff at St Christophers and has been adapted from work by the NCETM and incorporates the progression of mathematical areas suggested by White Rose and Power Maths. All teachers have been given the scheme of work from the White Rose Maths Hub (Part, Part Whole models) and are required to base their planning around their year group's modules and not to move onto a higher year group's scheme work. These modules use the Singapore Maths Methods (Bar Modelling), Part - Whole Model and are affiliated to the workings of the 2014 Maths Programme of Study. Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that a variety of resources are used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.

KEY STAGE 1 – YEAR 1 & YEAR 2
Children develop the core ideas that underpin all calculations. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.
Key language: whole, part, ones, ten, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus, less, more, group, share, equal, equals, is equal to, groups, equal groups, times, multiply, multiplied by, divide, share, shared equally, times-table

Addition and subtraction: Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations. A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 − 3 and 15 − 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.	**Multiplication and division:** Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division. They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.	**Fractions:** In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole. In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator.

KEY STAGE 2 – YEAR 3 & YEAR 4

In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, sharing, grouping, bar model

Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply.

In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns.

By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.

Multiplication and division: Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35.

Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively.

Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit.

Children develop column methods to support multiplications in these cases.

For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.

Fractions: Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount and develop this with the aid of a bar model and other representations alongside.

In Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1.

Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.

KEY STAGE 2 – YEAR 5 & 6

In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.

Key language: decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number

Addition and subtraction: Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage.

Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods.

Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

Multiplication and division: Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers.

Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000.

Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions.

Multiplication and division of decimals are also introduced and refined in Year 6.

Fractions: Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them.

Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic.

Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.

Addition

Addition- EYFS			
Objectives	**Concrete**	**Pictorial**	**Abstract**

Knows that a group of things change in quantity when something is added.

Find the total number of items in two groups by counting all of them.

Says the number that is one more than a given number.

Finds one more from a group of up to five objects, then ten objects.

Automatically recall number bonds for numbers 0–10.

In practical activities and discussion, beginning to use the vocabulary involved in adding.

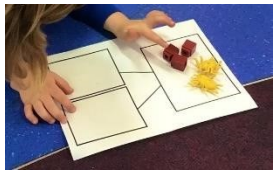
Using quantities and objects, they add two single digit numbers and count on to find the answer.

Solve problems including doubling

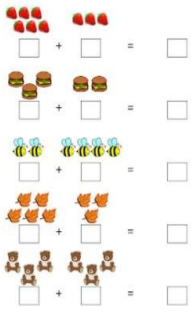
Link the number symbol (numeral) with its cardinal number value.

Use toys and general classroom resources for children to physically manipulate, group/regroup.

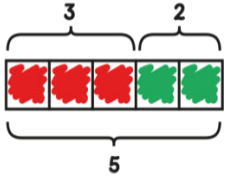
Use specific maths resources such as counters, snap cubes, Numicon etc.



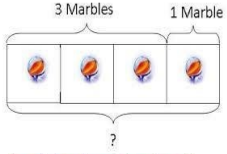
Use visual supports such as ten frames, part part whole and addition mats, with the physical objects and resources that can be manipulated.

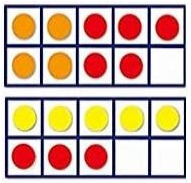


Two groups of pictures so children are able to count the total.

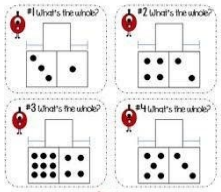


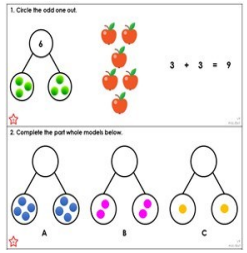
Bar model using visuals, pictures/icons or colours.





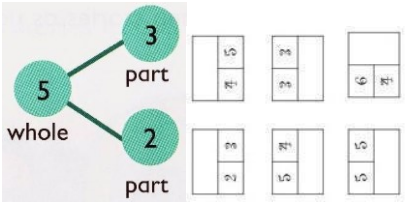
Use visual supports such as ten frames, part part whole and addition mats with pictures/icons.





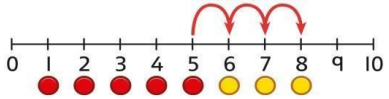
A focus on symbols and numbers to form a calculation.

$$5+2=7$$



Children use a number line to understand how to link counting on with finding one more., e.g. ‘One more than 6 is 7’ and ‘7 is one more than 6’.

Learn to link counting on with adding more than one.

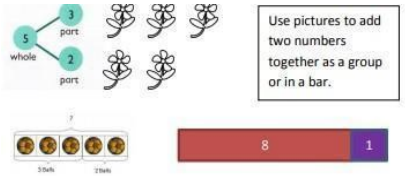
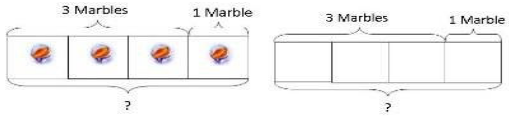
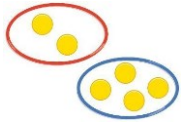
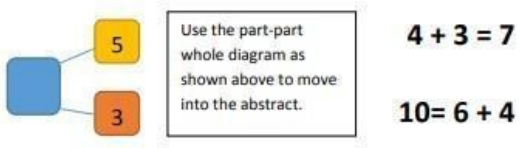
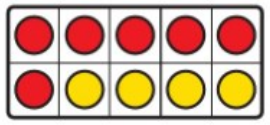
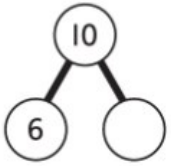


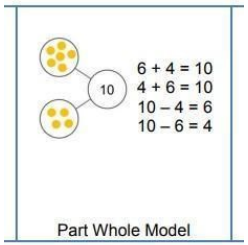
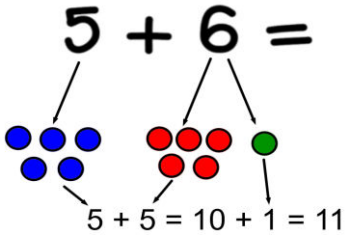
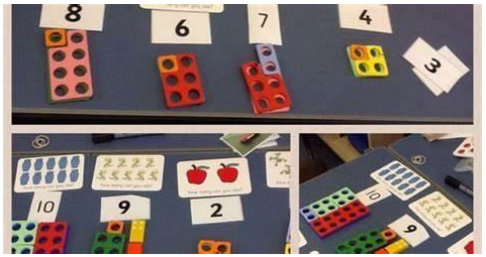
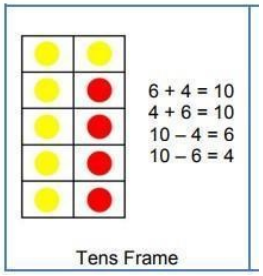
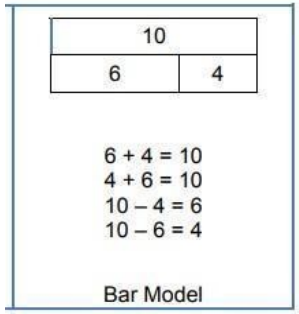
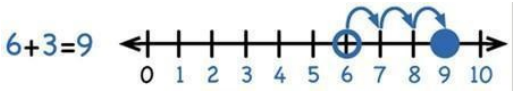
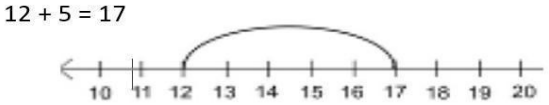
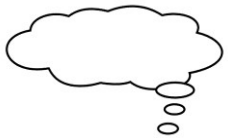
Children will begin to record a simple number sentence/addition calculation.

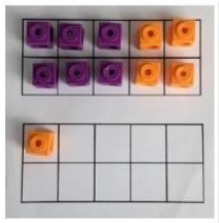
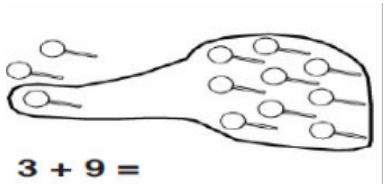
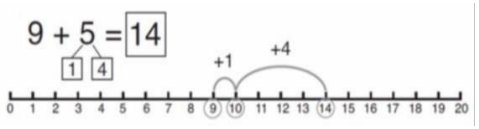
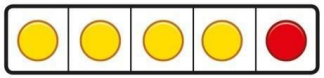
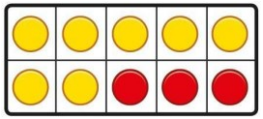
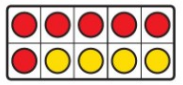
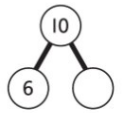
Vocabulary	altogether, add, more, plus, and, make, total, equal to, equals, double, most, count on, number line, part, whole.

Year 1

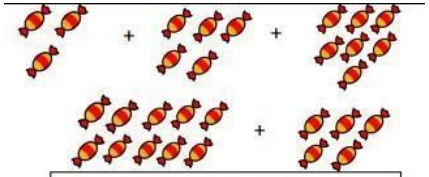
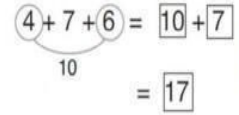
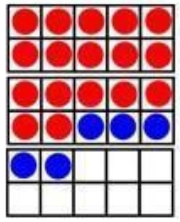
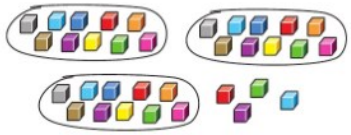
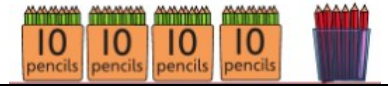
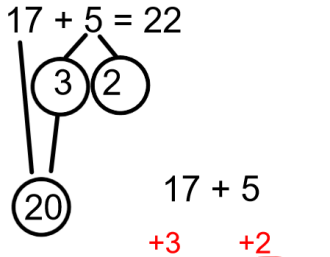
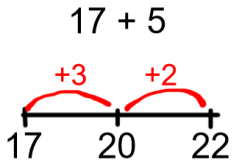
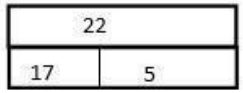
Children develop the core ideas that underpin all calculations. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

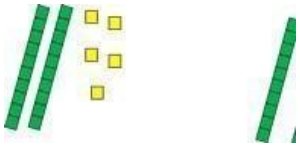
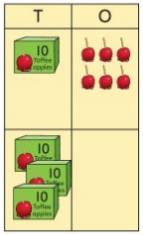
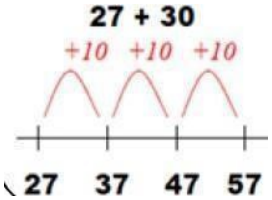
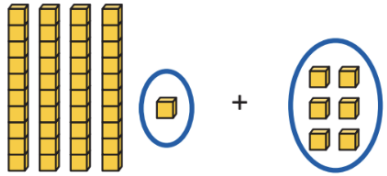
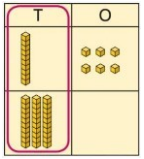
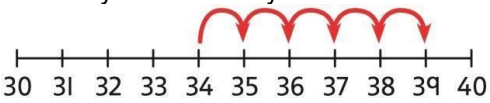
Objective and Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model Counting in near doubles	 Use cubes to add two numbers together as a group or in a bar. (Some children may still need to use real objects)  Use part-part whole model Sort people and objects into parts and understand the relationship with the whole. *The parts are 2 and 4. The whole is 6.*	 Children draw to represent the parts and understand the relationship with the whole. The Bar Model will be continued from EYFS as a method to support problem solving involving addition, continuing with the concrete representations and moving onto using pictorial representations of objects.  Some children will also move onto the abstract. Pictorial (concrete)  Abstract: *The parts are 2 and 4. The whole is 6.*	 Use a part-whole model to represent the numbers. Use a part-whole model alongside other representations to find number bonds.   Make sure to include examples where one of the parts is zero.

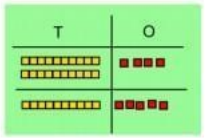
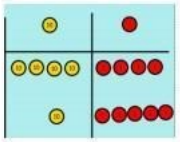
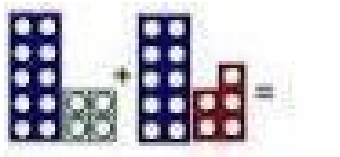
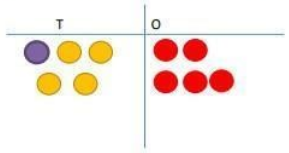
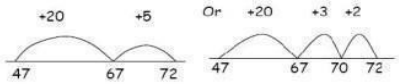
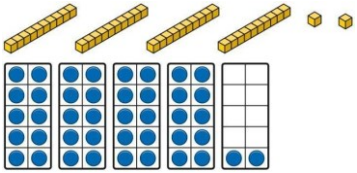
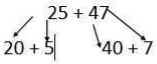
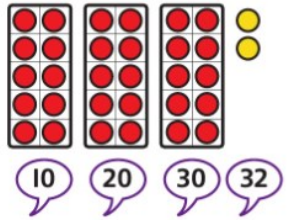
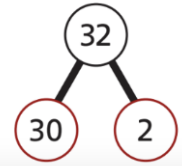
			
Represent and use number bonds and related subtraction facts within 20	 Use real objects	 (Some children may need to initially use real objects then move onto the representation, egg boxes may also be used to support this)	 Bar model and part-part whole to be used alongside abstract
Addition and subtraction of one-digit and two-digit numbers to 20 including 0.		 Start at the larger number on the number line and count on in ones.	Only refer to abstract once Concrete and Pictorial methods / understanding are secure: 5 + 12 = 17 17= 12 + 5
Start at the bigger number and counting on	Start with the larger number on the bead string and then count to the smaller number 1 by 1 to find the answer.	 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	Place the larger number in your head and count on the smaller number to find your answer. 

Regrouping to make 10 (The 'Make 10' strategy)	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames. Break apart a group and put back together to find and form number bonds.  *3 + 4 = 7*  *6 = 2 + 4*	 Use pictures or a number line. Regroup or Partition the smaller number using the part part  whole model to make 10. Use five and ten frames to represent key number bonds.  *5 = 4 + 1* *10* = 7 + 3 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now? Use a part-whole model alongside other representations to find number bonds.   Make sure to include examples where one of the parts is zero.
Vocabulary	**add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, part, whole**		

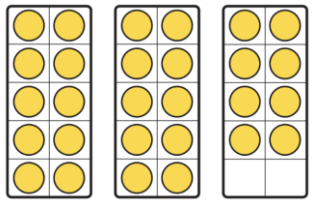
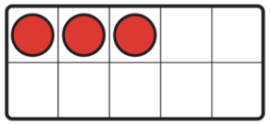
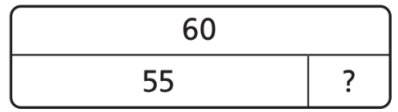
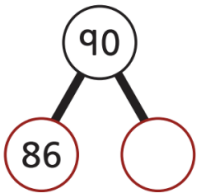
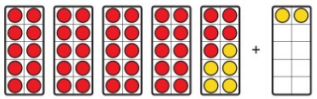
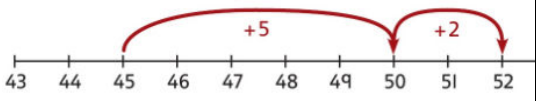
Addition- Year 2

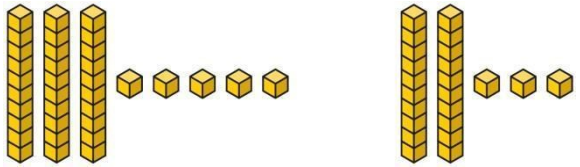
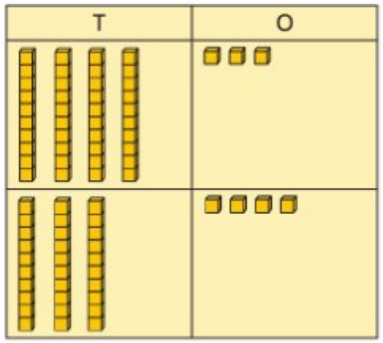
Objective and Strategy	Concrete	Pictorial	Abstract
Adding 3 1-digit numbers	**4 + 7 + 6= 17** Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Adding a 2-digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32 Group objects into 10s and 1s.  Bundle straws, pencils or pens to understand unitising of 10s. 	Use part / whole and number line model.   Bar Model	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 

Adding a 2-digit number and multiples of 10	 25 + 10 = 35 Explore that the ones digit does not change Add the 1s to find the total. Use known bonds within 10.  *41 is 4 tens and 1 one.* *41 add 6 ones is 4 tens and 7 ones.* Add the 10s using a place value grid to support, using classroom items to represent the numbers.  *16 is 1 ten and 6 ones.* *30 is 3 tens.* *There are 4 tens and 6 ones in total.*	 Base 10 may be used above the number line initially. The calculation will be shown alongside the number line to see the connection Add the ones using known bonds  *1 + 6 = 7* *So* *41 + 6 = 47* Add the 10s using a place value grid to support.  *16 is 1 ten and 6 ones.* *30 is 3 tens.* *There are 4 tens and 6 ones in total.*	27 + 10 = 37 27 + 20 = 47 27 + □ = 57 Add the 1s. Understand the link between counting on and using known number facts. Children should be encouraged to use known number bonds to improve efficiency and accuracy.  *4 + 5 = 9* *So* *34 + 5 = 39* Use known bonds and knowledge of place value to add multiples of 10 *16 + 30 = ?* *So 1 ten + 3 tens is 4 tens* *There are 4 tens and 6 ones in total.* *16 + 30 = 46* *Count on in tens from a given number* *'Start on 16', '26', '36', '46'* *16 + 30 = 46*

Adding two 2-digit numbers (No regrouping)	**24 + 15=** Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.   (Some children may not be ready for place value counters in Y2) Numicon may also be used 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.   Use number line and bridge ten using part whole if necessary. Base 10 may be used above the number line. The calculation will be shown alongside the number line to see the connection **Model** / **Calculation** <u>The Bar Model</u> (Singapore maths) will be used to support problem solving moving onto the generalisation that b+c=a. Children will focus on using the abstract representation with the pictorial to support where necessary. Understand 10s and 1s equipment, and link with visual representations on ten frames.  Represent numbers on a place value grid, using equipment or numerals.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72 Partitioning: *Recording addition in columns supports place value and prepares for formal written methods with larger numbers.* Toward the end of the year, children move to more formal recording using partitioning method: 40 + 7 30 + 5 70 + 12 Partition 2-digit numbers into 10s and 1s  

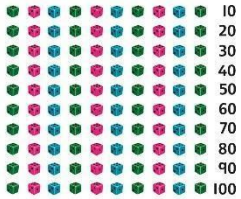
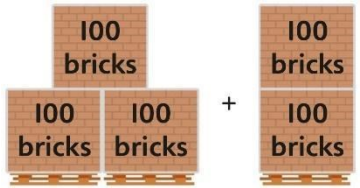
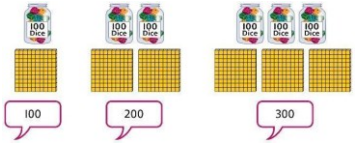
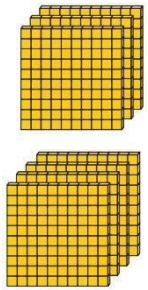
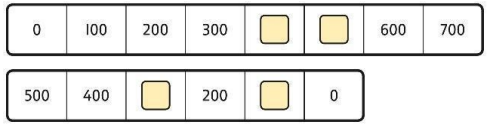
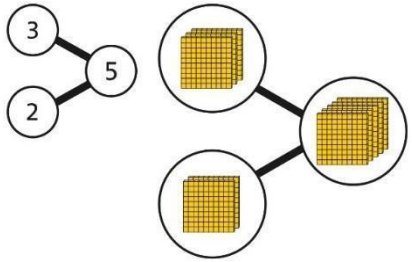
			32 = 30 + 2

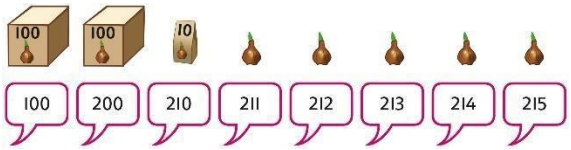
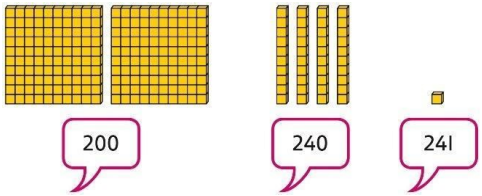
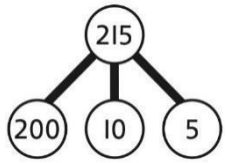
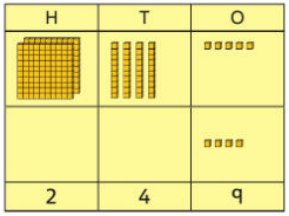
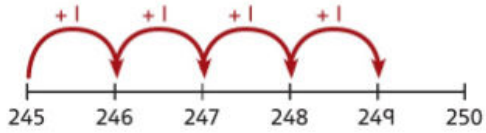
Add to the next 10	Use known bonds to 10 to add to the next multiple of 10  *8 + 2 = 10* *So* *28 + 2 = 30*	Use known bonds to 10 to add to the next multiple of 10 3 + ☐ = 10 33 + ☐ = 40 43 + ☐ = 50 73 + ☐ = 80 	Use known bonds to 10 to add to the next multiple of 10  55 + ☐ = 60  86 + ☐ = 90
Add across a 10	Use place value equipment to support adding across any multiple of 10  *45 + 5 + 2 = 52* *45 + 7 = 52*	Add across any multiple of 10 using two jumps  *45 + 5 + 2 = 52* *45 + 7 = 52*	Add across any multiple of 10 using two steps 45 + 5 + 2 = 52 45 + 7 = 52

Add the 1s and 10s separately	Add the 10s and 1s separately.  *5 + 3 = 8* *There are 8 ones in total.* *3 + 2 = 5* *There are 5 tens in total.* *35 + 23 = 58*	Add the 1s and the 10s then recombine  3 ones and 4 ones is 7 ones 4 tens and 3 tens is 7 tens 43 + 34 = 77	Add the 10s and 1s separately. 32 + 11 *30 + 10 = 40* *2 + 1 = 3* *32 + 11 = 43*
Vocabulary	**add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary**		

Addition- Year 3

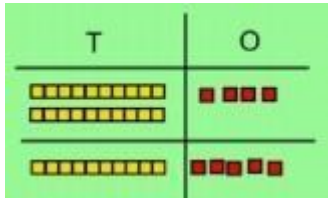
In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

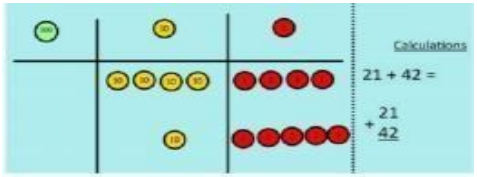
Objective and Strategy	Concrete	Pictorial	Abstract
Understanding 100s Adding 100s	Understand the cardinality of 100, and the link with 10 tens. Use cubes to place into groups of 10 tens.  Use known facts and unitising to add multiples of 100.  *3 + 2 = 5* *3 hundreds + 2 hundreds = 5 hundreds* *300 + 200 = 500*	Unitise 100 and count in steps of 100.  Use known facts and unitising to add multiples of 100.  *3 + 4 = 7* *3 hundreds + 4 hundreds = 7 hundreds* *300 + 400 = 700*	Represent steps of 100 on a number line and a number track and count up to 1,000 and back to 0.  Use known facts and unitising to add multiples of 100. Represent the addition on a number line. Use a part-whole model to support unitising.  *3 + 2 = 5* *300 + 200 = 500*

Understanding place value to 1,000	Unitise 100s, 10s and 1s to build 3-digit numbers.  	Use equipment to represent numbers to 1,000.  Use a place value grid to support the structure of numbers to 1,000. Place value counters are used alongside other equipment. Children should understand how each counter represents a different unitised amount.	Represent the parts of numbers to 1,000 using a part-whole model.  *215 = 200 + 10 + 5* Recognise numbers to 1,000 represented on a number line, including those between intervals.
3-digit number + 1s, no exchange or bridging	Use number bonds to add the 1s.  *214 + 4 = ?* *Now there are 4 + 4 ones in total.* *4 + 4 = 8* *214 + 4 = 218*	Use number bonds to add the 1s.  *245 + 4* *5 + 4 = 9* *245 + 4 = 249*	Understand the link with counting on. *245 + 4*  Use number bonds to add the 1s and understand that this is more efficient and less prone to error. *245 + 4 = ?* *I will add the 1s.* *5 + 4 = 9* *So, 245 + 4 = 249*

Add and subtract numbers with up to 3-digits, using formal written methods of columnar addition

Column addition (no exchange / regrouping)



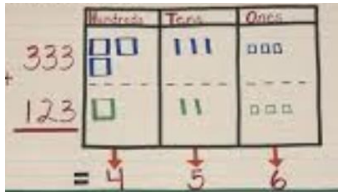


hundreds tens ones.
Using manipulatives (dienes, numicon, counters), children are to line up.
Children should be secure with using PV counters before moving onto pictorial.
The calculation will be shown alongside the model used to see the connection

Model	Calculation

Calculate mentally by forming the number bond for the 10s.

234 + 50 There are 3 tens and 5 tens altogether.
3 + 5 = 8 In total there are 8 tens.
234 + 50 = 284



Children are to draw, in a PV frame, the manipulatives, that they are using.

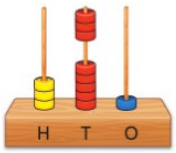
Secure knowledge of representation with the PV columns.

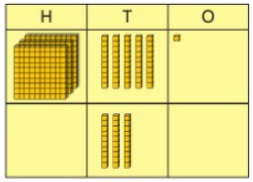
The calculation will be shown alongside the model to see the connection

Model	Calculation

Calculate mentally by forming the number bond for the 10s.

351 + 30 = ?





5 tens + 3 tens = 8 tens
351 + 30 = 381

$$\begin{array}{r} 223 \\ +\ 114 \\ \hline 337 \end{array}$$

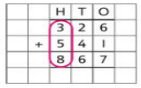
Children to move onto recording more formally.
Some children may need to use the expanded method (see below).

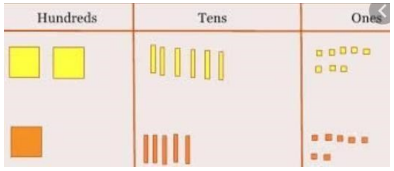
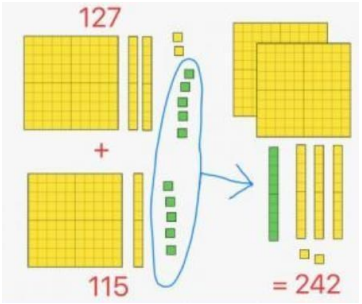
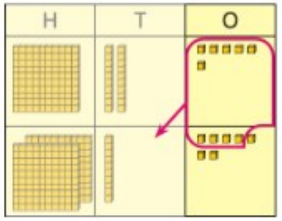
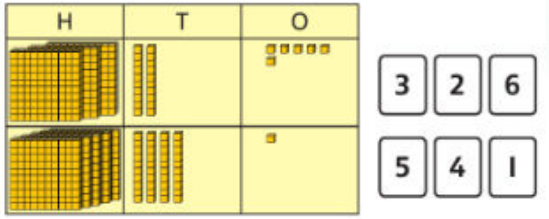
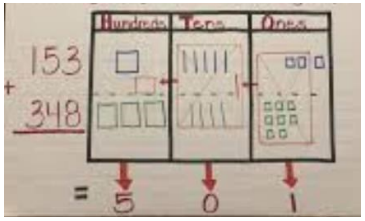
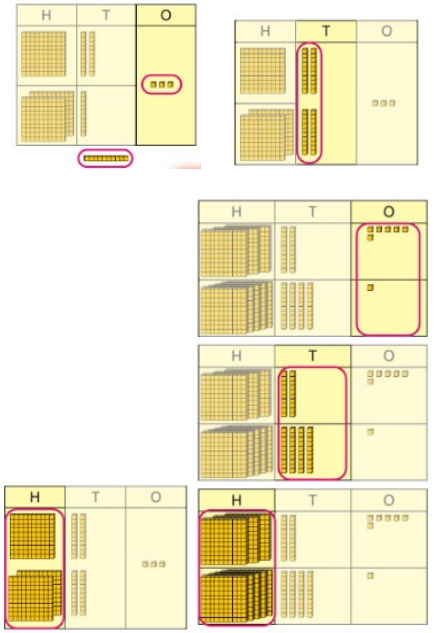
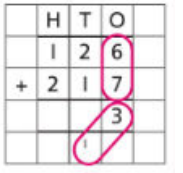
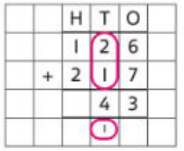
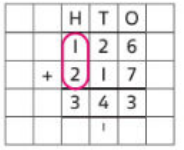
$$\begin{array}{rcrl} 20 & + & 5 & \\ 40 & + & 8 & \\ \hline 60 & + & 13 & = 73 \end{array}$$

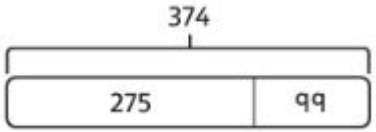
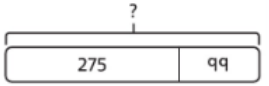
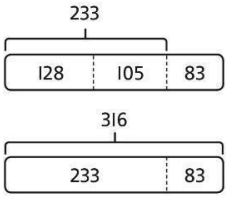
Children are to begin with the abstract: expanded form.
For those children, that are confident after AFL, the below method should be used.

$$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ 11 \end{array}$$

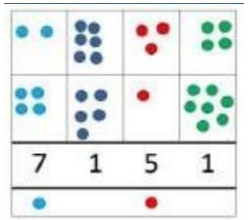
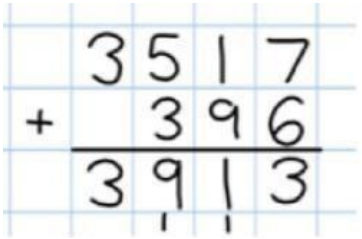
Use a column method to solve efficiently, using known bonds.
Children must understand how this relates to place value at every stage of the calculation.

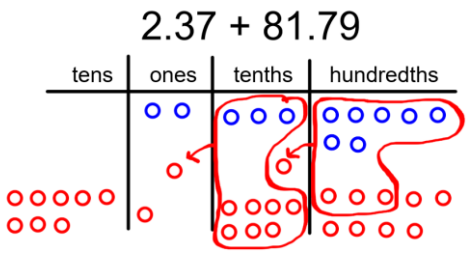
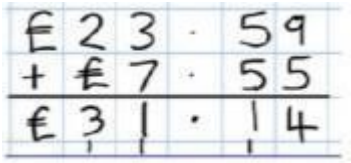


Column addition (exchange / regrouping)	  Exchange ten ones for a ten. Model using Dienes, Numicon and place value counters. *326 + 541 is represented as:* Use place value equipment to enact the exchange required.  *There are 13 ones.* *I will exchange 10 ones for 1 ten.* 	 Children can draw a representation of the grid to further support their understanding, carrying the ten ***underneath*** the line. Represent the place value grid with equipment to model the stages of column addition. Model the stages of column addition using place value equipment on a place value grid. 	Use column addition, ensuring understanding of place value at every stage of the calculation. Use the vertical column method to represent the addition. Children must understand how this relates to place value at each stage of the calculation.    *126 + 217 = 343* *Note:* Children should also study examples where exchange is required in more than one column, for example *185 + 318 = ?*

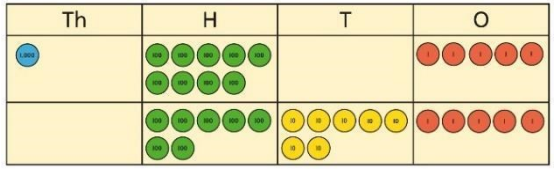
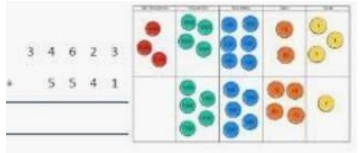
Representing addition problems, and selecting appropriate methods	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. *275 + 99 = ?*  *275 + 99 = 374*	Use representations to support choices of appropriate methods.  *I will add 100, then subtract 1 to find the solution.* *128 + 105 + 83 = ?* *I need to add three numbers.* 128 + 105 = 233 
Vocabulary	**addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary**		

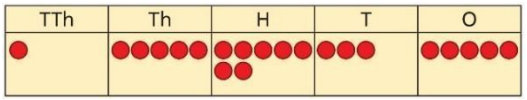
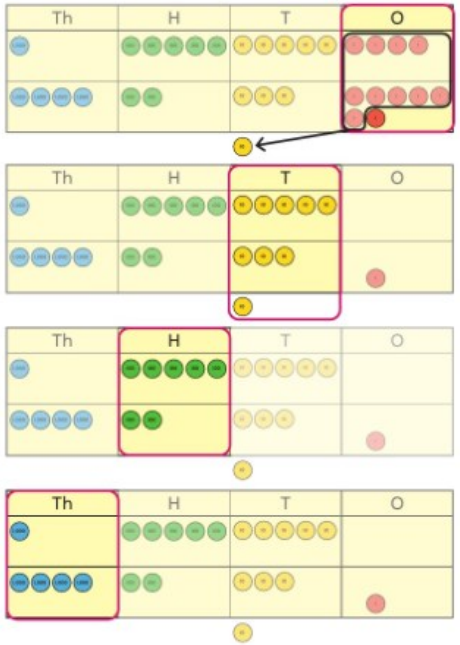
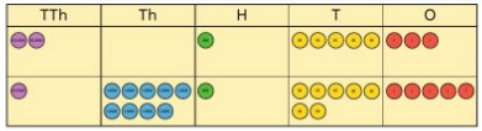
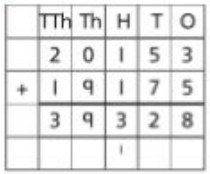
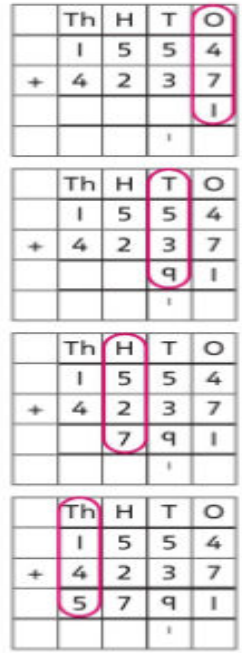
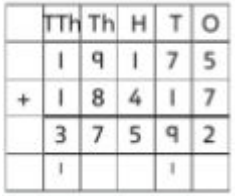
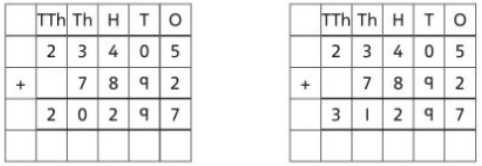
Addition- Year 4

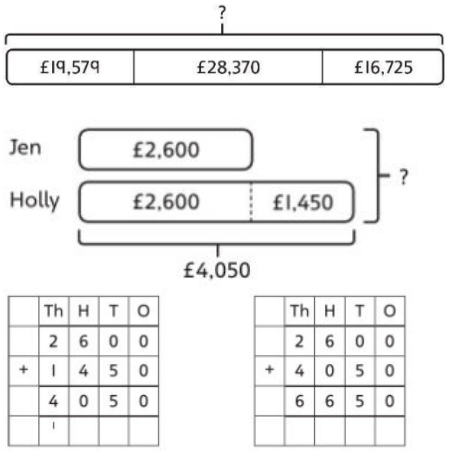
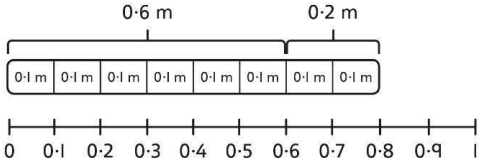
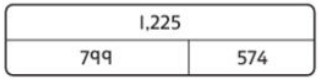
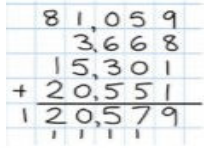
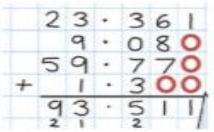
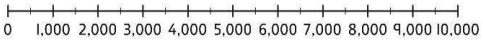
Objective and Strategy	Concrete	Pictorial	Abstract
Using formal written methods of columnar addition where appropriate add numbers with up to 4 digits (with exchange)	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.  The calculation will be shown alongside the manipulative used to see the connection	 Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.	 Continue from previous work to carry hundreds as well as tens.

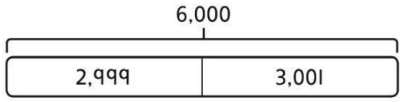
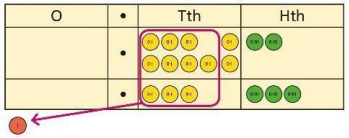
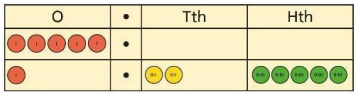
		Model	Calculation		
Add decimals with 2 decimal places, including money.	 Introduce decimal place value counters and model exchange for addition.				 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.
Vocabulary	**addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point**				

Addition- Year 5 (Begin teaching Year 6 strategies from Spring term)

Objective and Strategy	Concrete	Pictorial	Abstract
add numbers with more than 4 digits.	Use place value equipment on a place value grid to organise thinking. Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers. *Use equipment.to show 1,905 + 775.* 	Use place value equipment to model required exchanges.	 Children should have abstract supported by a pictorial or concrete if needed. Use a column method to add, including exchanges.

	Why have only three columns been used for the second row? Why is the Thousands box empty? *Which columns will total 10 or more?* Use place value equipment to represent additions. .  *Add a row of counters onto the place value grid to show 15,735 + 4,012*	 Include examples that exchange in more than one column. Represent additions, using place value equipment on a place value grid alongside written methods.  *I need to exchange 10 tens for a 100.*  Bar models represent addition of two or more numbers in the context of problem solving.	 Include examples that exchange in more than one column. Use column addition, including exchanges.  Use approximation to check whether answers are reasonable.  *I will use 23,000 + 8,000 to check.*

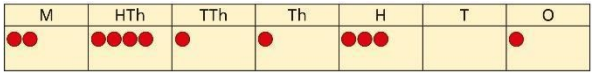
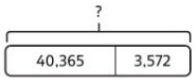
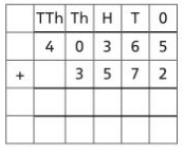
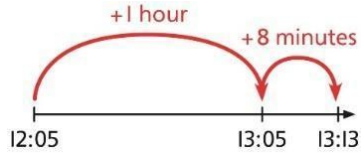
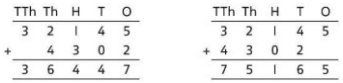
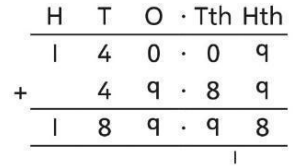
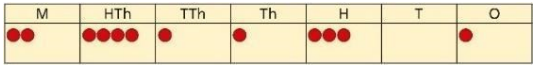
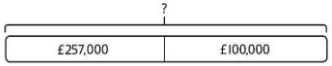
		 Th H T O 2 6 0 0 + 1 4 5 0 4 0 5 0 1 Th H T O 2 6 0 0 + 4 0 5 0 6 6 5 0	
Add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.	Link measure with addition of decimals. *Two lengths of fencing are 0·6 m and 0·2 m.* *How long are they when added together?*  *Show 0·23 + 0·45 using place value counters.*	Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate. Use a bar model with a number line to add tenths.  *0·6 + 0·2 = 0·8* *6 tenths + 2 tenths = 8 tenths*  Th H T O 7 9 9 + 5 7 4 1 3 7 3 1 1 1	  Insert zeros for place holders. Use rounding and estimating on a number line to check the reasonableness of an addition.  *912 + 6,149 = ?* *I used rounding to work out that the answer should be approximately: 1,000 + 6,000 = 7,000.* Understand the link with adding fractions. $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$

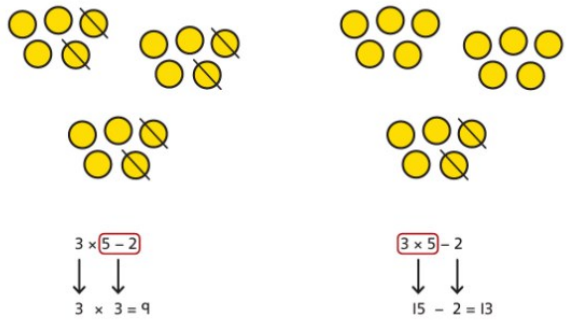
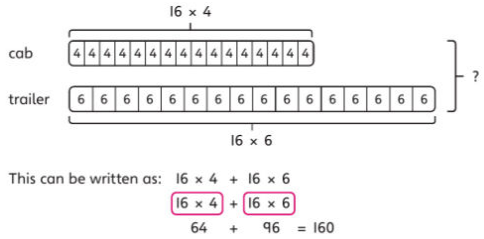
		I chose to work out 574 + 800, then subtract 1.  *This is equivalent to 3,000 + 3,000.* Represent exchange where necessary.  O · Tth Hth 0 · 9 2 + 0 · 3 3 1 · 2 5 Include examples where the numbers of decimal places are different.  O · Tth Hth 5 · 0 0 + 1 · 2 5 6 · 2 5	*6 tenths + 2 tenths = 8 tenths* *0·6 + 0·2 = 0·8* Add using a column method, ensuring that children understand the link with place value. O · Tth Hth 0 · 2 3 + 0 · 4 5 0 · 6 8 Include exchange where required, alongside an understanding of place value. O · Tth Hth 0 · 9 2 + 0 · 3 3 1 · 2 5 Include additions where the numbers of decimal places are different. O · Tth Hth 3 · 4 0 + 0 · 6 5 · *3.4 + 0.65 = ?*

Vocabulary - Previous addition, add, more, and make, sum, total, altogether Efficient written method, Column addition and subtraction, total Thousand more/less than Negative integers Count through zero Roman numerals (I to C) hundredths, tenths, ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions

Addition- Year 6

	Concrete	Pictorial	Abstract

Comparing and selecting efficient methods for addition	Represent 7-digit numbers on a place value grid and use this to support thinking and mental methods.  	Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.   Use bar model and number line representations to model addition in problem-solving and measure contexts.  	Use column addition where mental methods are not efficient. Recognise common errors with column addition. *32,145 + 4,302 = ?*  *Which method has been completed accurately?* *What mistake has been made?* Column methods are also used for decimal additions where mental methods are not efficient. 
Selecting mental methods for larger numbers where appropriate	Represent 7-digit numbers on a place value grid and use this to support thinking and mental methods.  *2,411,301 + 500,000 = ?* *This would be 5 more counters in the HTh place.* *So, the total is 2,911,301.* *2,411,301 + 500,000 = 2,911,301*	Use a bar model to support thinking in addition problems. *257,000 + 99,000 = ?*  *I added 100 thousands then subtracted 1 thousand.* *257 thousands + 100 thousands = 357 thousands* *257,000 + 100,000 = 357,000* *357,000 – 1,000 = 356,000* *So, 257,000 + 99,000 = 356,000*	Use place value and unitising to support mental calculations with larger numbers. *195,000 + 6,000 = ?* *195 + 5 + 1 = 201* *195 thousands + 6 thousands = 201 thousands* *So, 195,000 + 6,000 = 201,000*

Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. *3 × 5 − 2 = ?* 	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. 	Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. *4 + 6 × 16* *4 + 96 = 100* *(4 + 6) × 16* *10 × 16 = 160*
Vocabulary	**addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point, order of operation, thousands, hundredths, tenths, ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions, ten millions**		

Subtraction

Subtraction- EYFS			
Objectives	**Concrete**	**Pictorial**	**Abstract**

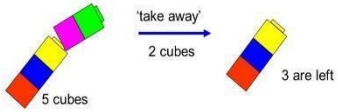
Knows that a group of things change in quantity when something is taken away

Find one less from a group of five objects, then ten objects.

In practical activities and discussion, beginning to use the vocabulary involved in subtracting.

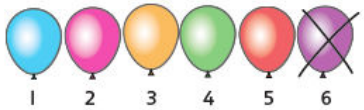
Using quantities and objects, they subtract two single digit numbers and count back to find the answer.

Use toys and general classroom resources for children to physically manipulate, group/regroup.



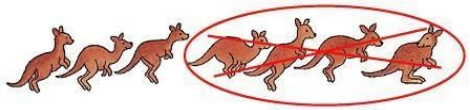
Use specific maths resources such as snap cubes, Numicon, bead strings etc.

Children arrange objects and remove to find how many are left.

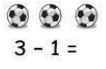


1 less than 6 is 5.
6 subtract 1 is 5.

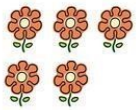
Children separate a whole into parts and understand how one part can be found by subtraction.



6 - 4 =



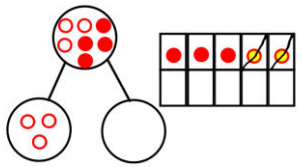
3 - 1 =

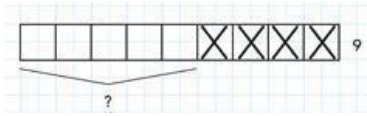


5 - 3 =

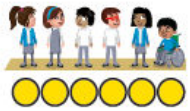
7 - 2 =

A group of pictures for children to cross out or cover quantities to subtraction





Children draw and cross out or use counters to represent objects from a problem.



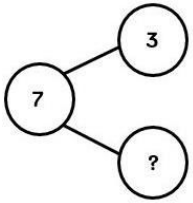
Now there are 6 children.

A focus on symbols and numbers to form a calculation.

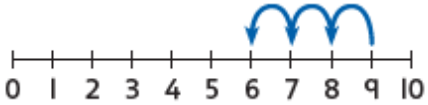
10 - 6 = 4

3	?
7	

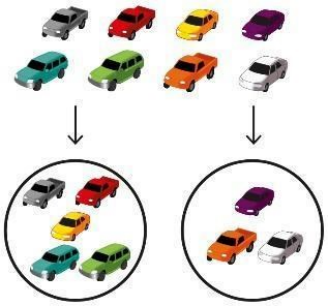
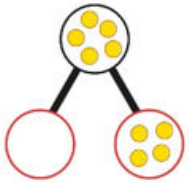
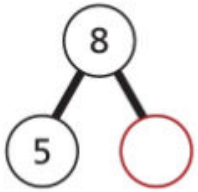
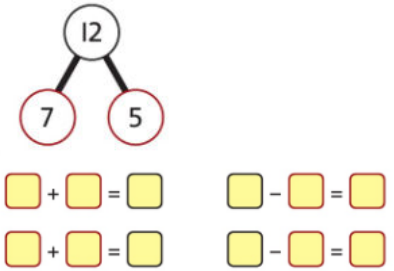
7 - 3 = ?



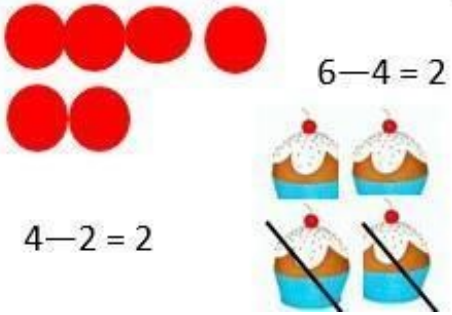
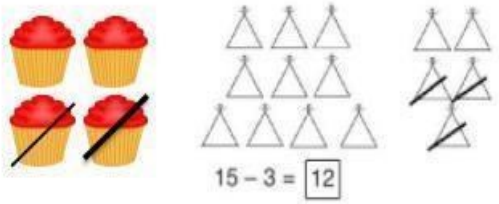
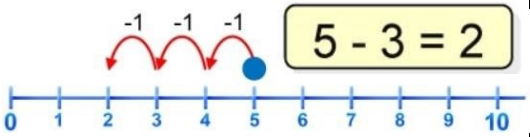
Children count back to take away and use a number line or number track to support the method.

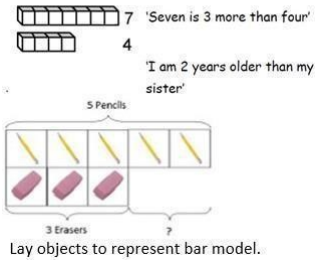
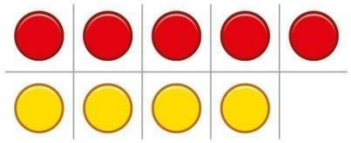
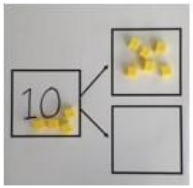
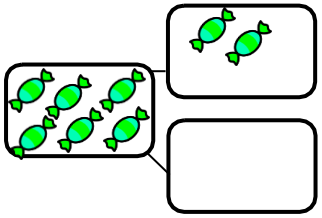
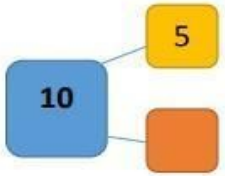


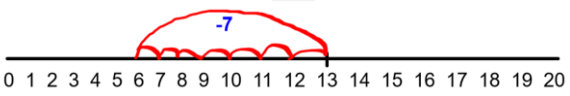
9 – 3 = 6

	 *8 – 5 = ?*   Use visual supports such as ten frames, part part whole and subtraction mats, with the physical objects and resources that can be manipulated.	Children represent a whole and a part and understand how to find the missing part by subtraction.   Use visual supports such as ten frames, part part whole and bar model with pictures/icons.	Children use a part-whole model to support the subtraction to find a missing part.  *8 – 5 = ?* Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model.  * No expectation for children to be able to record a number sentence/addition calculation.
Vocabulary	**equal to, take, take-away, less, minus, subtract, how many more, how many fewer/less than, most, least count back, how many left, how much less is**		

Subtraction- Year 1

Objective and Strategy	Concrete	Pictorial	Abstract
Subtract one-digit and two-digit numbers to 20, including 0. Taking away ones	Use physical objects, counters , cubes etc to show how objects can be taken away.  	Cross out drawn objects to show what has been taken away.  	Ensure Concrete and Pictorial methods are secure before teaching Abstract 7—4 = 3 16—9 = 7
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. **13 – 4**  Use counters and move them away from the group as you take then away counting backwards as you go. 	 Count back on a number line or track. Start at the bigger number and count back the smaller number showing the jumps on the number line.	Put 13 in your head, count back 4. What number are you at? (Use your fingers to help you)

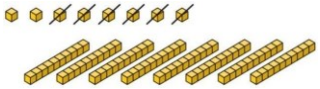
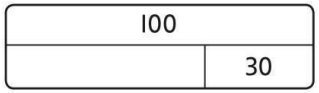
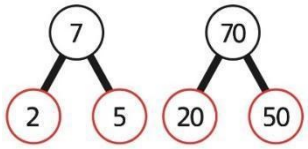
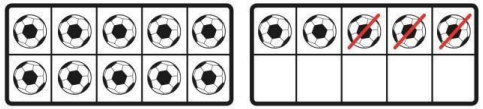
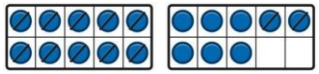
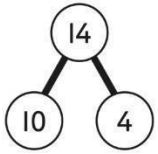
Find the difference	Compare objects and amounts  Lay objects to represent bar model. Arrange two groups so that the difference between the groups can be worked out.  *8 is 2 more than 6.* *6 is 2 less than 8.* *The difference between 8 and 6 is 2*	Represent objects using sketches or counters to support finding the difference.  *5 − 4 = 1* *The difference between 5 and 4 is 1.*	Hannah has12 sweets and her sister has 5. How many **MORE** sweet does Hannah have than her sister? Children understand 'find the difference' as subtraction.  *10 − 4 = 6* *The difference between 10 and 6 is 4.*
Represent and use number bonds and related subtraction facts within 20 Part-part whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use a pictorial representation of objects to show the part-part whole model	 Move to using numbers within the part whole model.

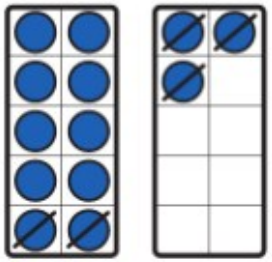
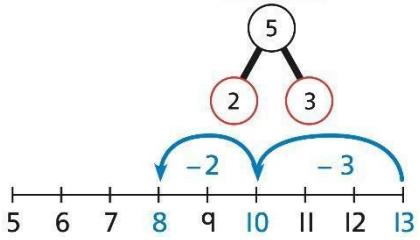
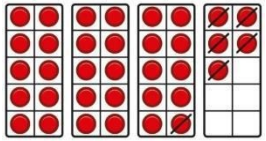
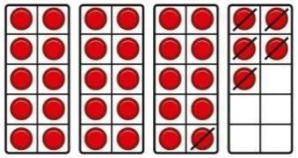
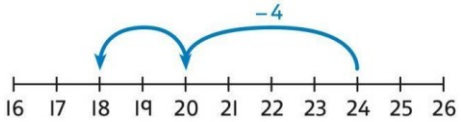
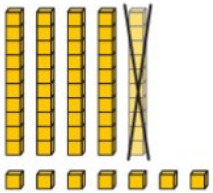
<table>
<tr>
<td>Make 10</td>
<td>14 – 9 =
 
Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</td>
<td>13 - 7 = 6
 

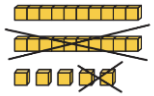
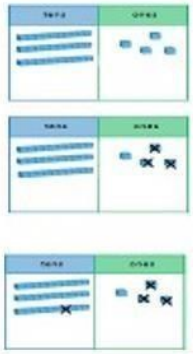
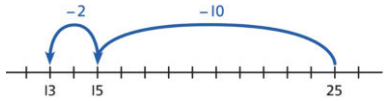
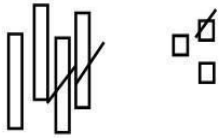
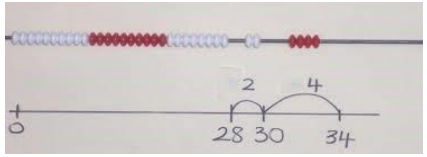
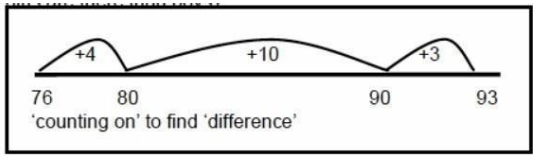
Start at 13. Take away 7 altogether.</td>
<td>16 – 8=

How many will we take off to reach 10?
How many do we have left to take off?</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…</td>
</tr>
</table>

Subtraction- Year 2

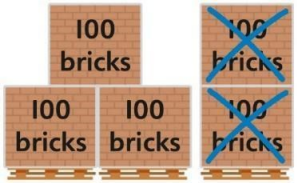
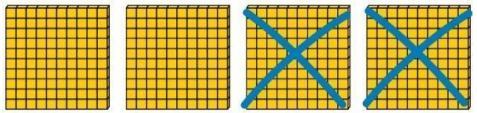
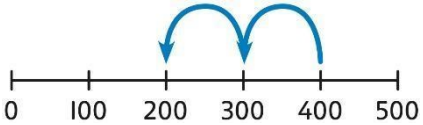
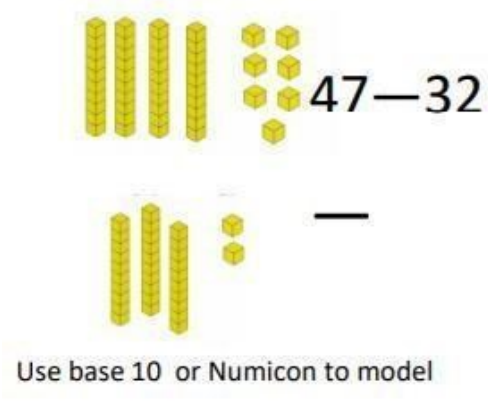
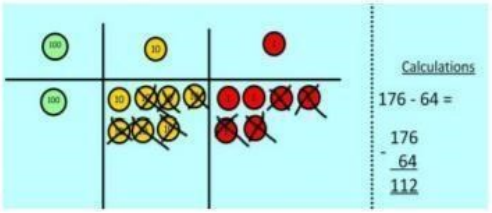
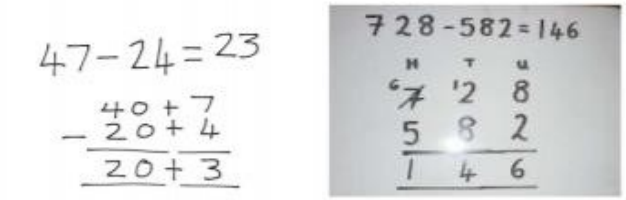
Objective and Strategy	Concrete	Pictorial	Abstract
Subtract two multiples of 10	Use known number bonds and unitising to subtract multiples of 10.  *8 subtract 6 is 2.* *So, 8 tens subtract 6 tens is 2 tens.*	Use known number bonds and unitising to subtract multiples of 10.  *10 − 3 = 7* *So, 10 tens subtract 3 tens is 7 tens.*	Use known number bonds and unitising to subtract multiples of 10.  *7 tens subtract 5 tens is 2 tens.* *70 − 50 = 20*
Subtraction within 20	**Subtraction within 20** Understand when and how to subtract 1s efficiently.  *5 − 3 = 2* *15 − 3 = 12*	**Subtraction within 20** Understand how to use knowledge of bonds within 10 to subtract efficiently. *5 − 3 = 2* *15 − 3 = 12*	**Subtraction within 20** Understand when and how to subtract 1s efficiently. Use a bead string to subtract 1s efficiently.  *5 − 3 = 2* *15 − 3 = 12*
Subtracting 10s and 1s	**Subtracting 10s and 1s** For example: 18 − 12 Use ten frames to represent the efficient method of subtracting 12.  *First subtract the 10, then subtract 2.*	**Subtracting 10s and 1s** Use a part-whole model to support the calculation.  *19 − 14* *9 − 4 = 5* *19 − 10 = 9* *So, 19 − 14 = 5*	**Subtracting 10s and 1s** For example: 18 − 12 *First subtract the 10, then take away 2.*

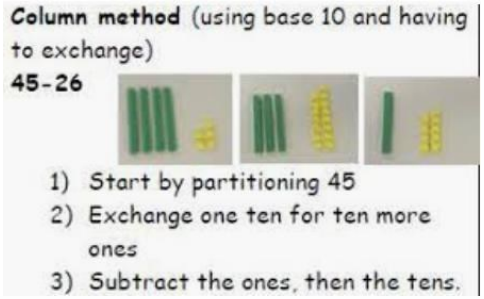
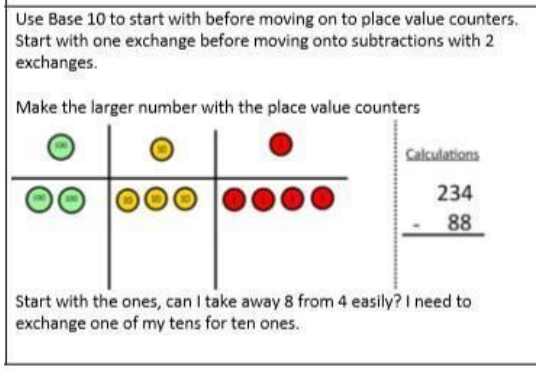
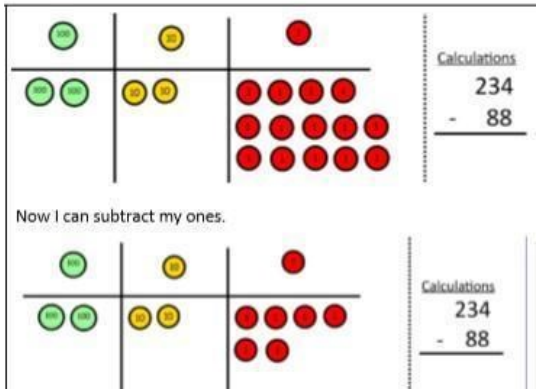
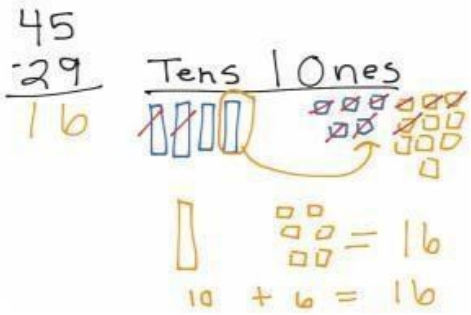
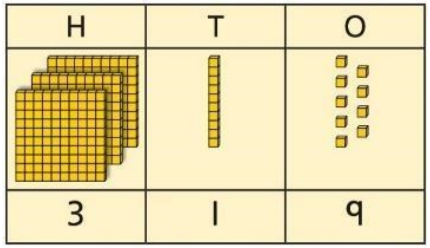
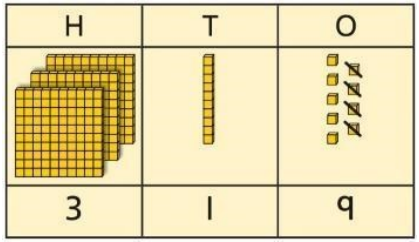
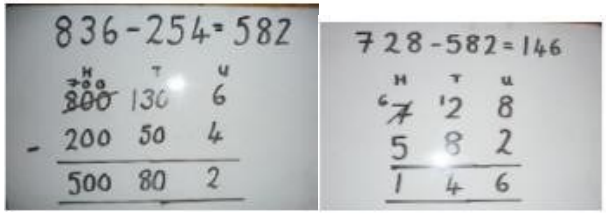
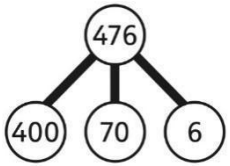
Subtraction bridging 10 using number bonds .	**Subtraction bridging 10 using number bonds** Represent the use of bonds using ten frames.  *For 13 – 5, I take away 3 to make 10, then take away 2 to make 8.*	**Subtraction bridging 10 using number bonds** Use a number line and a part-whole model to support the method. *13 − 5* 	**Subtraction bridging 10 using number bonds** For example: 12 − 7 Arrange objects into a 10 and some 1s, then decide on how to split the 7 into parts. *7 is 2 and 5, so I take away the 2 and then the 5.*
Subtracting a single-digit number bridging 10	Bridge 10 by using known bonds.  *35 − 6* *I took away 5 counters, then 1 more.*	Bridge 10 by using known bonds.  *35 − 6* *First, I will subtract 5, then 1.*	Bridge 10 by using known bonds.  *24 − 6 = ?* *24 − 4 − 2 = ?*
Subtract tens from a 2-digit number		Subtract tens using known bonds  57 – 10 = 47	Subtract tens using known bonds *43 – 10 = 33*

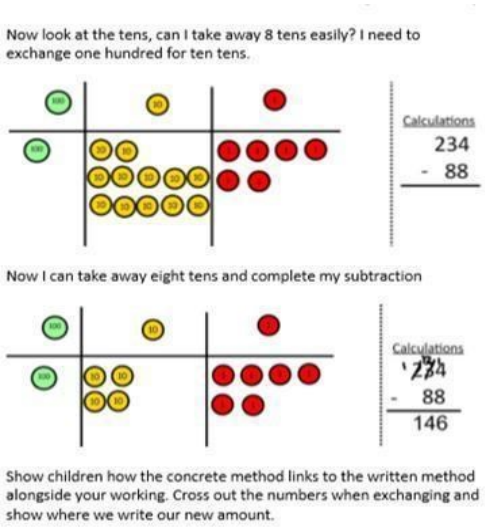
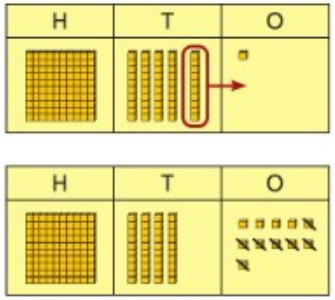
Subtract a two-digit number and ones, a two digit number and tens, two two-digit numbers Partitioning to subtract without regrouping: *'Friendly numbers'*	Subtract 10s then 1s using place value equipment.  *25 – 10 – 2 = 13* *25 – 12 = 13* 34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Subtract 10s then 1s with a number line for visual support.  *25 – 10 – 2 = 13* *25 – 12 = 13* Children draw representations of Dienes and cross off.  43—21 = 22	Subtract 10s then 1s. *25 – 10 – 2 = 13* *25 – 12 = 13* 43—21 = 22 *Recording subtraction in columns supports place value and prepares for formal written methods with larger numbers.* Toward the end of the year, children move to more formal recording using partitioning method: e.g. 43-21=22 40 and 3 -20 and 1 20 and 2
Make ten strategy	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to the next ten and then the rest.	93—76 = 17

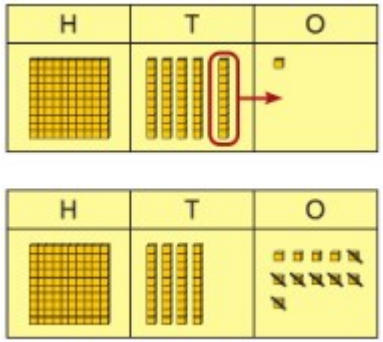
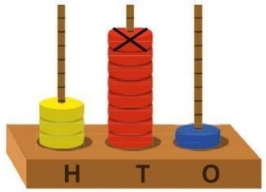
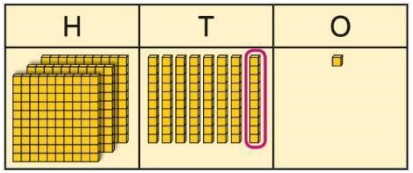
Vocabulary	**equal to, take, take-away, less, minus, how many more, how many fewer/less than, most, least, how many left, how much less is… count on, strategy, partition, tens, ones**

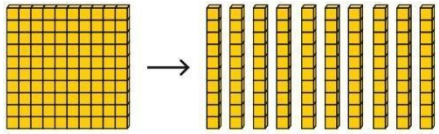
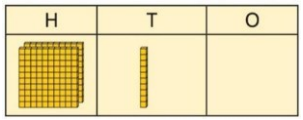
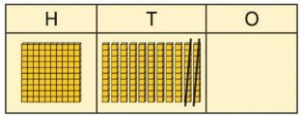
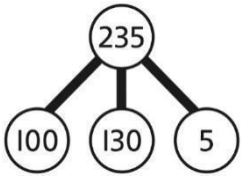
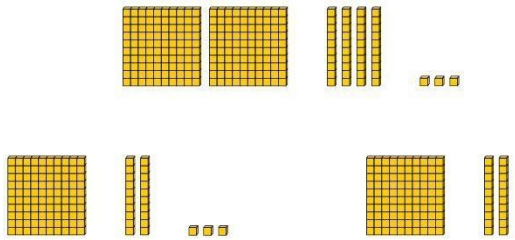
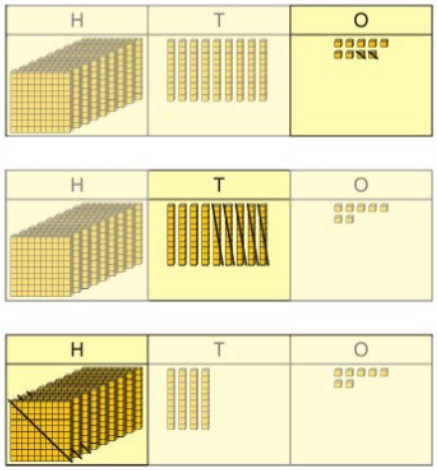
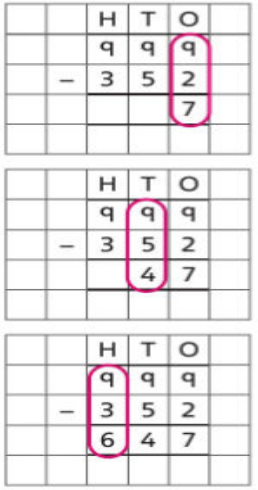
Subtraction- Year 3

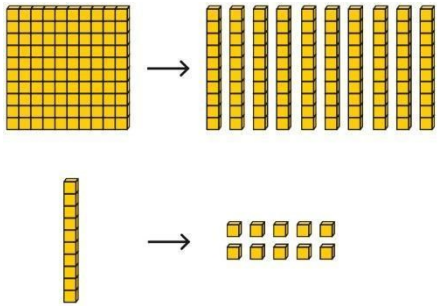
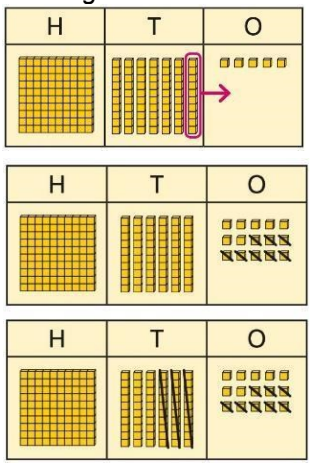
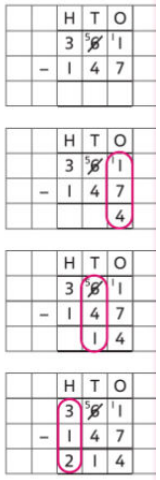
Objective and Strategy	Concrete	Pictorial	Abstract
Subtracting 100s	Use known facts and unitising to subtract multiples of 100.  *5 − 2 = 3* *500 − 200 = 300*	Use known facts and unitising to subtract multiples of 100.  *4 − 2 = 2* *400 − 200 = 200*	Understand the link with counting back in 100s.  *400 − 200 = 200* Use known facts and unitising as efficient and accurate methods. *I know that 7 − 4 = 3. Therefore, I know that 700 − 400 = 300.*
To subtract numbers with up to three-digits, using formal written methods of columnar subtraction Column subtraction (without exchanging)	 The calculation will be shown alongside the model chosen to see the connection **Model** | **Calculation**	Children are to be secure with use of PV counters before moving onto abstract. 	Children should begin with the expanded form. Moving onto a more formal way as below. 

Column Subtraction (with exchanging)	Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.  **Column method** (using base 10 and having to exchange) **45-26** 1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens.  Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters Calculations 234 - 88 Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Calculations 234 - 88 Now I can subtract my ones. Calculations 234 - 88	 45 -29 16 Tens | Ones = 16 10 + 6 = 16 When confident, children can find their own way to record the exchange/regrouping Use number bonds to subtract the 1s.  H T O 3 I 9 *319 − 4 = ?*  H T O 3 I 9 *9 − 4 = 5* *319 − 4 = 315*	Children should begin with the expanded form. Moving onto a more formal way as below (bottom picture).  836-254=582 800 130 6 - 200 50 4 500 80 2 728-582=146 6 7 12 8 5 8 2 1 4 6 Understand the link with counting back using a number line. Use known number bonds to calculate mentally. *476 − 4 = ?* *6 − 4 = 2* *476 − 4 = 472*  476 400 70 6

3-digit number − 1s, exchange or bridging required	Understand why an exchange is necessary by exploring why 1 ten must be exchanged. Use place value equipment.  Use number bonds to subtract the 1s.  *214 − 3 = ?*  *4 − 3 = 1* *214 − 3 = 211*	Represent the required exchange on a place value grid. *151 − 7 = ?* 	Calculate mentally by using known bonds. *151 − 7 = ?* *151 − 1 − 6 = 144*

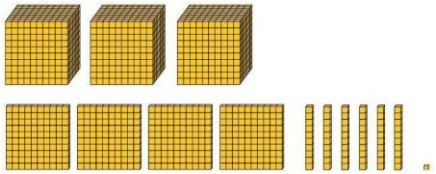
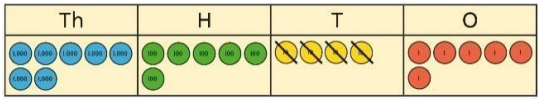
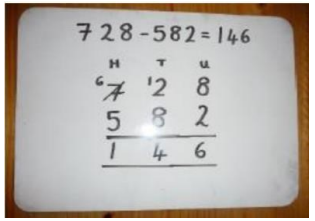
3-digit number − 1s, exchange or bridging required	Understand why an exchange is necessary by exploring why 1 ten must be exchanged. Use place value equipment.	Represent the required exchange on a place value grid. *151 − 7 = ?* 	Calculate mentally by using known bonds. *151 − 7 = ?* *151 − 1 − 6 = 144*
3-digit number − 10s, no exchange	Subtract the 10s using known bonds.  *381 − 10 = ?* *8 tens with 1 removed is 7 tens.* *381 − 10 = 371*	Subtract the 10s using known bonds.  *8 tens − 1 ten = 7 tens* *381 − 10 = 371*	Use known bonds to subtract the 10s mentally. *372 − 50 = ?* *70 − 50 = 20* *So, 372 − 50 = 322*

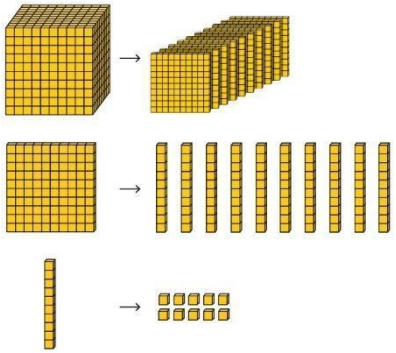
3-digit number − 10s, exchange or bridging required	Use equipment to understand the exchange of 1 hundred for 10 tens. 	Represent the exchange on a place value grid using equipment. *210 − 20 = ?*  *I need to exchange 1 hundred for 10 tens, to help subtract 2 tens.*  *210 − 20 = 190*	Understand the link with counting back on a number line. Use flexible partitioning to support the calculation. *235 − 60 = ?*  *235 = 100 + 130 + 5* *235 − 60 = 100 + 70 + 5* *= 175*
3-digit number − up to 3-digit number	Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away. 	Represent the calculation on a place value grid.  	Use column subtraction to calculate accurately and efficiently. 

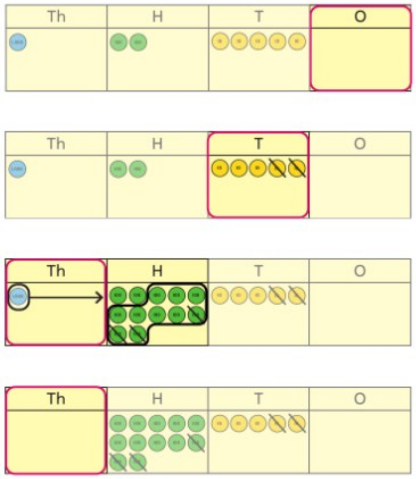
3-digit number – up to 3-digit number, exchange required	Use base 10 equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones. 	*Model the required exchange on a place value grid.* *175 – 38 = ?* *I need to subtract 8 ones, so I will exchange a ten for 10 ones.* 	Use column subtraction to work accurately and efficiently.  If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly. Children should also understand how to exchange in calculations where there is a zero in the 10s column.

Vocabulary	**equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is…difference, count on, strategy, partition, hundreds, tens, ones**

Subtraction- Year 4

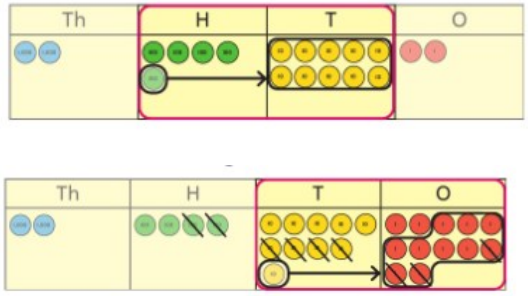
Objective and Strategy	Concrete	Pictorial	Abstract
Choosing mental methods where appropriate	Use place value equipment to justify mental methods.  *What number will be left if we take away 300?*	Use place value grids to support mental methods where appropriate.  *7,646 − 40 = 7,606*	Use knowledge of place value and unitising to subtract mentally where appropriate. *3,501 − 2,000* *3 thousands − 2 thousands = 1 thousand* *3,501 − 2,000 = 1,501*
Subtract numbers with up to 4 digits using the formal written methods appropriate of columnar subtraction where appropriate Year 4 subtraction with up to 4 digits.	Model process of exchange using Numicon, base ten and then move to PV counters. Use the phrase 'take and make' for exchange- see Y3 The calculation will be shown alongside the model chosen to see the connection Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.	Children to draw pv counters and show their exchange—see Y3 The calculation will be shown alongside the model chosen to see the connection **Model** / **Calculation** Represent place value equipment on a place value grid to subtract, including exchanges where needed.	 This will lead to an understanding of subtracting any number including decimals. Use column subtraction, with understanding of the place value of any exchange required. Make exchanges across more than one column where there is a zero as a place holder. *2,502 − 243 = ?*



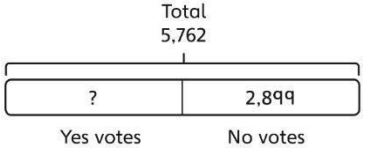


Make exchanges across more than one column where there is a zero as a place holder.

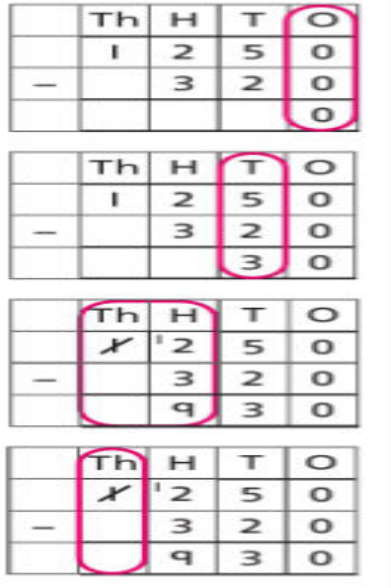
2,502 − 243 = ?



Use bar models to represent subtractions where a part needs to be calculated.

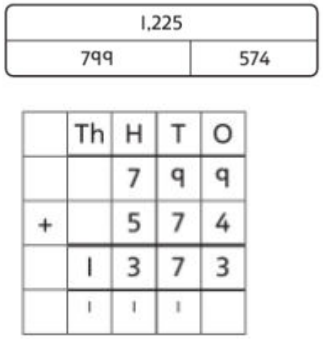


I can work out the total number of Yes votes using 5,762 − 2,899.

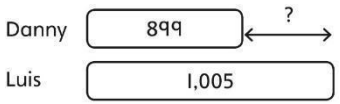
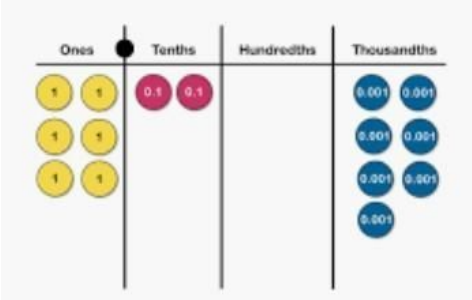
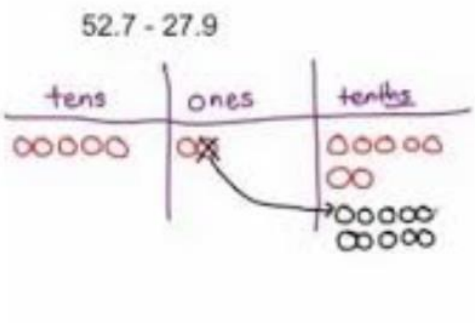
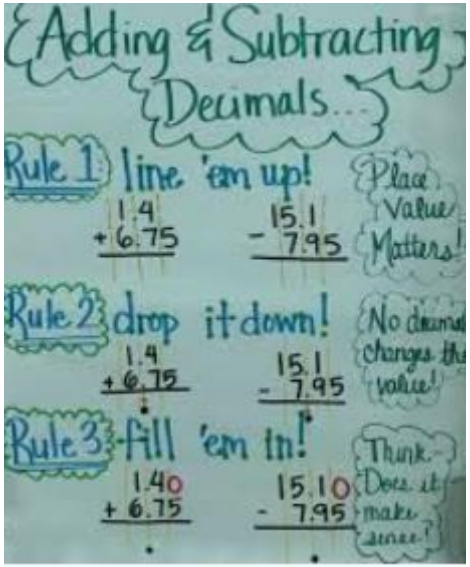


Use inverse operations to check subtractions.

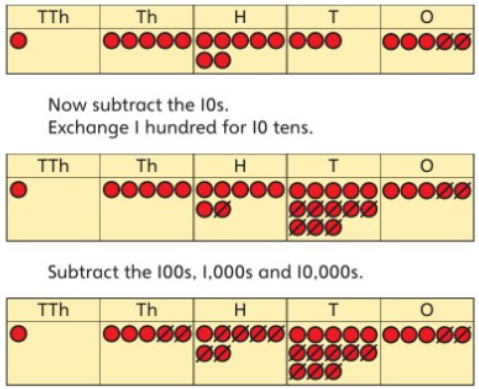
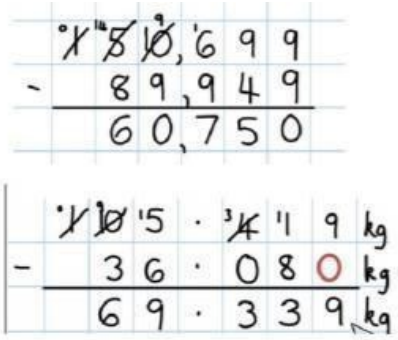
I calculated 1,225 − 799 = 574.
I will check by adding the parts.

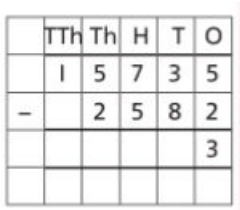
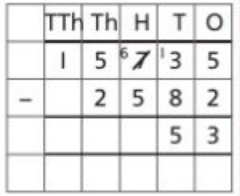
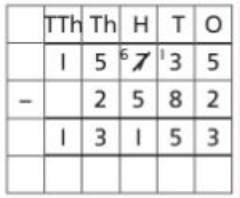
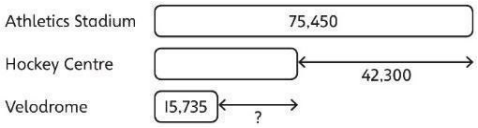
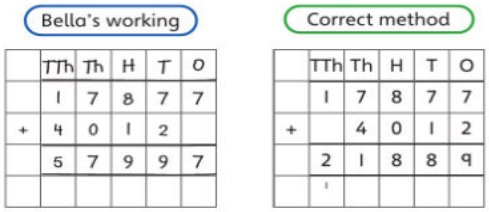
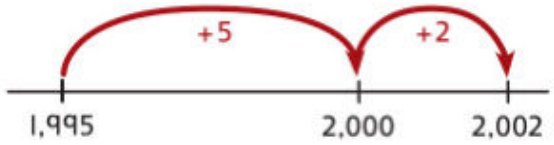


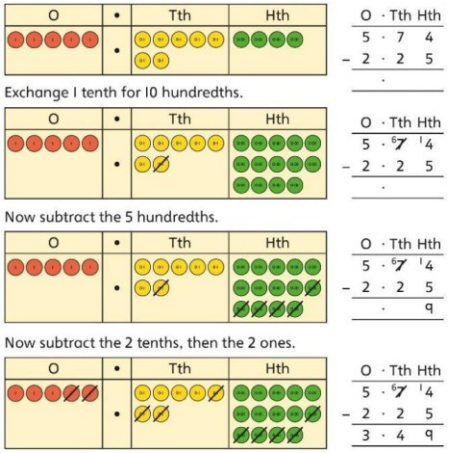
The parts do not add to make 1,225.
I must have made a mistake.

		Bar models can also represent 'find the difference' as a subtraction problem. 	
Introduce decimal subtraction through context of money	Children to be encourgaed to use counters to represent numbers and take counters away to subtract. 	 When confident, children can find their own way to record the exchange/regrouping	
Vocabulary	**equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is…difference, count on, strategy, partition, thousands, hundreds, tens, ones, tenths**		

Subtraction- Year 5 (Begin teaching Year 6 strategies from Spring term)

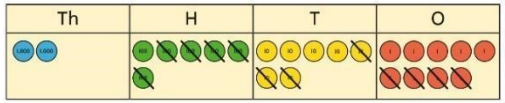
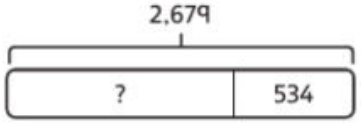
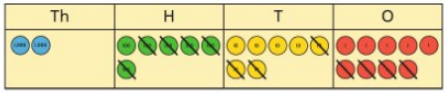
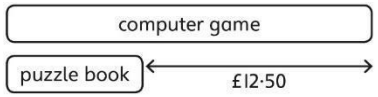
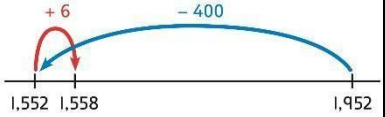
Objective and Strategy	Concrete	Pictorial	Abstract
Subtract with at least 4 digits, including money and measures. Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	Use place value equipment to understand where exchanges are required. *2,250 – 1,070 = ?*	Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required. *15,735 − 2,582 = 13,153* 	 Use column subtraction methods with exchange where required. TTh Th H T O 5 8 1 2 5 9 7 − 1 8 0 3 4 4 4 5 6 3

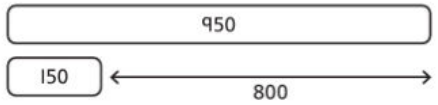
		   Bar models represent subtractions in problem contexts, including ‘find the difference’. 	*62,597 − 18,034 = 44,563* Children can explain the mistake made when the columns have not been ordered correctly. Use approximation to check calculations. Bella's working / Correct method  *I calculated 18,000 + 4,000 mentally to check my subtraction* To subtract two large numbers that are close, children find the difference by counting on. *2,002 − 1,995 = ?*  Use addition to check subtractions. *I calculated 7,546 − 2,355 = 5,191.* *I will check using the inverse.*

Subtracting decimals	Explore complements to a whole number by working in the context of length.  1 m – ☐ m = ☐ m *1 − 0·49 = ?*	Use a place value grid to represent the stages of column subtraction, including exchanges where required. *5·74 − 2·25 = ?*  	Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. *3·921 − 3·75 = ?* O · Tth Hth Thth 3 · 9 2 1 – 3 · 7 5 0

Vocabulary - how many more to make ...? ,how many more is ... than ...? ,how much more is ...?, subtract, take away
how many are left/left over? one less, two less, ten less ... one hundred less, how many fewer is ... than ...?, how much less is ...?, difference between

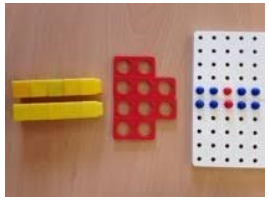
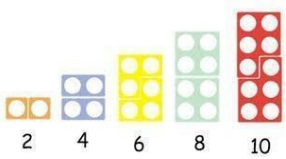
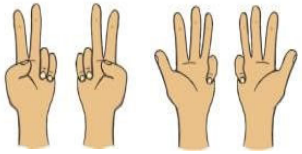
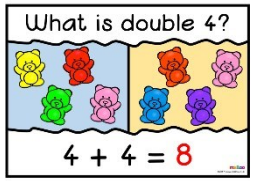
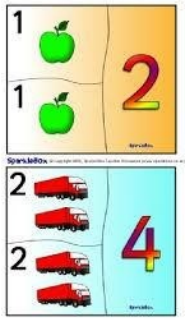
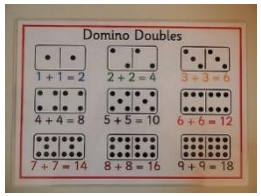
Subtraction- Year 6

Objective and Strategy	Concrete	Pictorial	Abstract
Comparing and selecting efficient methods	Use counters on a place value grid to represent subtractions of larger numbers. 	Compare subtraction methods alongside place value representations.  Th H T O 2 6 7 9 − 5 3 4 2 1 4 5  Use a bar model to represent calculations, including 'find the difference' with two bars as comparison. 	Compare and select methods. Use column subtraction when mental methods are not efficient. Use two different methods for one calculation as a checking strategy. Th H T O 1 ⁸~~9~~ ¹⁴~~5~~ ¹2 − 1 5 5 8 3 9 4  Use column subtraction for decimal problems, including in the context of measure. H T O · Tth Hth 3 0 9 · 6 0 − 2 0 6 · 4 0 1 0 3 · 2 0
Subtracting mentally with larger numbers		Use a bar model to show how unitising can support mental calculations. *950,000 − 150,000* *That is 950 thousands − 150 thousands*	Subtract efficiently from powers of 10. *10,000 − 500 = ?*

		 *So, the difference is 800 thousands.* *950,000 − 150,000 = 800,000*	
Vocabulary	**equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens units**		

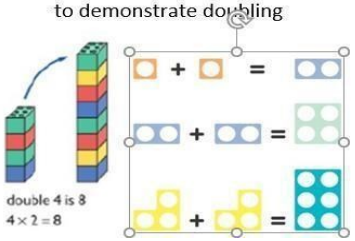
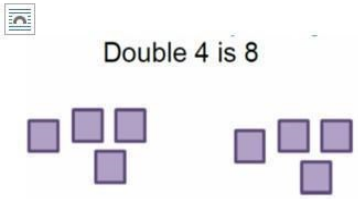
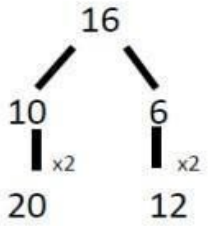
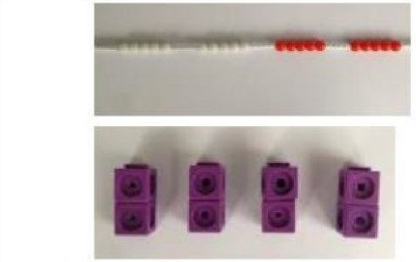
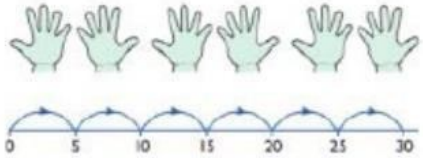
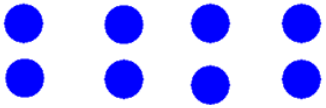
Multiplication

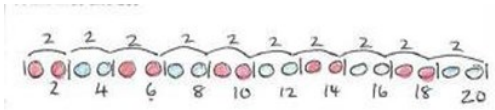
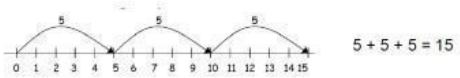
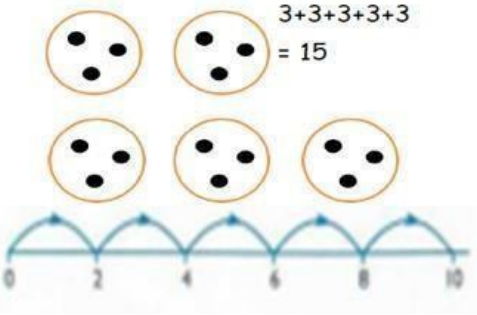
Multiplication-EYFS

Objectives	Concrete	Pictorial	Abstract
- Solve problems including doubling	   Counting and other maths resources for children to make 2 equal groups.  Physical and real life examples that encourage  children to see the concept of doubling as adding two equal groups.	   Pictures and icons that encourage children to see concept of doubling as adding two equal groups.	<table><tr><td>1+1=</td><td>7+7=</td></tr><tr><td>2+2=</td><td>8+8=</td></tr><tr><td>3+3=</td><td>9+9=</td></tr><tr><td>4+4=</td><td>10+10=</td></tr><tr><td>5+5=</td><td>11+11=</td></tr><tr><td>6+6=</td><td>12+12=</td></tr></table> Addition calculations to model adding two equal groups.

Vocabulary	Groups of, lots of, altogether, equal to, doubles

Multiplication- Year 1

Objective and Strategy	Concrete	Pictorial	Abstract
Doubling Counting in near doubles	Use practical activities using manip-ultives including cubes and Numicon to demonstrate doubling  	Draw pictures to show how to double numbers  	Partition a number and then double each part before recombining it back together. 
Counting in multiples	 Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.  Children make representations to show counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. **2, 4, 6, 8, 10** **5, 10, 15, 20, 25, 30**

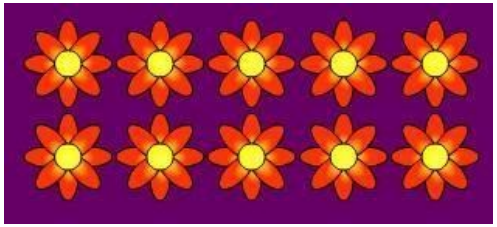
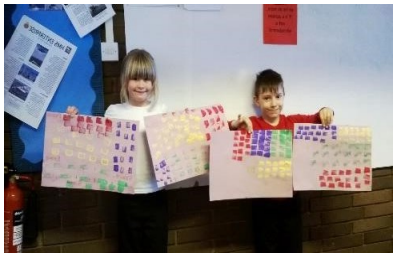
			
Repeated addition	 Use different objects to add equal groups.	 2 add 2 add 2 equals 6  5 + 5 + 5 = 15 Use pictorial including number lines to solve problems There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 3+3+3+3+3 = 15 	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

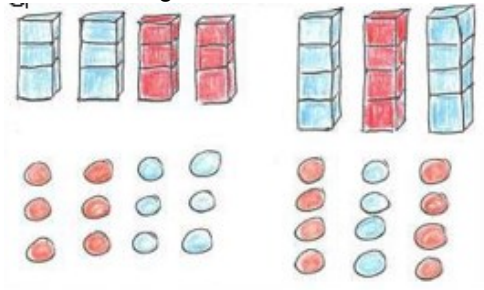
<table>
<tr>
<td>Making equal groups and counting the total</td>
<td> 
Use manipulatives to create equal groups.</td>
<td> 
2+2+2+2=8</td>
<td>4 equal groups of 2 = 8
3 equal groups of 2 = 6

There are 7 equal groups of 5 counters.

How many counters are there altogether?

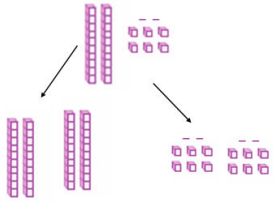
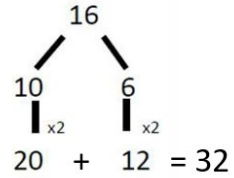
There are ☐ counters altogether.</td>
</tr>
<tr>
<td>Understanding arrays</td>
<td>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.

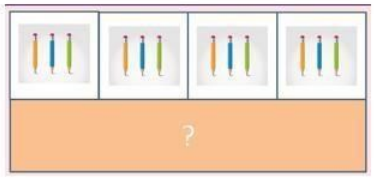
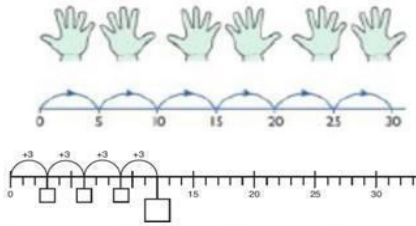
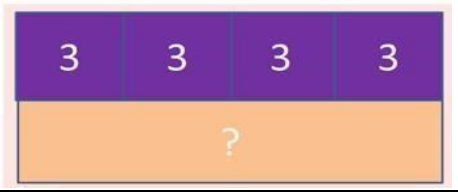
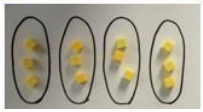
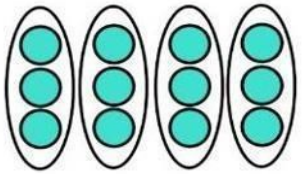
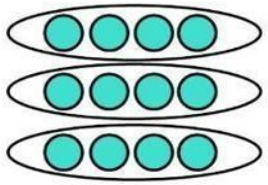
  </td>
<td> 

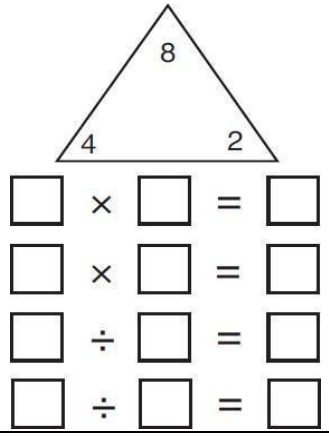
Draw representations of arrays to show understanding.
  </td>
<td>Begin to show repeated addition sentences alongside multiplication, e.g.

3 + 3 as 3 x 2 = 6
2 + 2 + 2 as 2 x 3 = 6

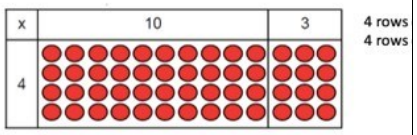
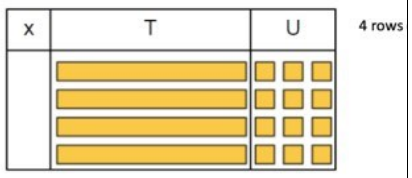
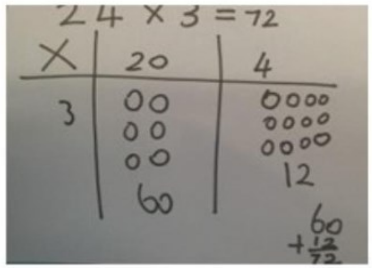
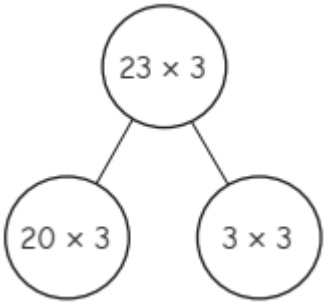
2 x 5 = 10</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">Groups of, lots of, times, array, altogether, multiply, equal, multiple, repeated addition, doubles</td>
</tr>
</table>

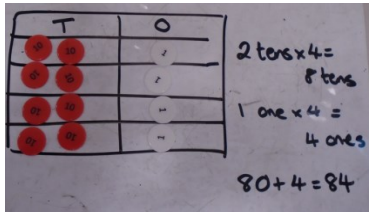
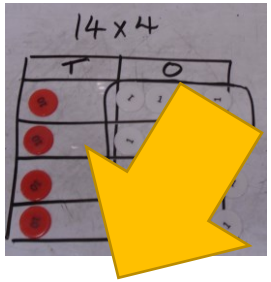
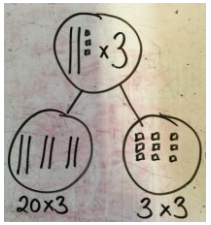
Multiplication- Year 2			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Doubling	Model doubling using dienes and PV counters.  	Draw pictures and representations to show how to double numbers.	Partition a number and then double each part before recombining it back together. 

Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	**Ensure secure understanding of counting/ repeated addition before embarking on learning times tables.** $12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.
Vocabulary	**Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.**		

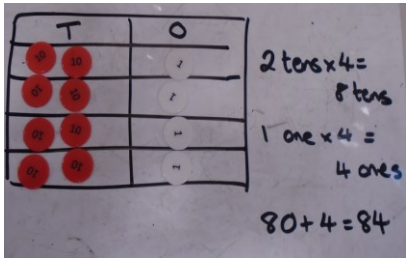
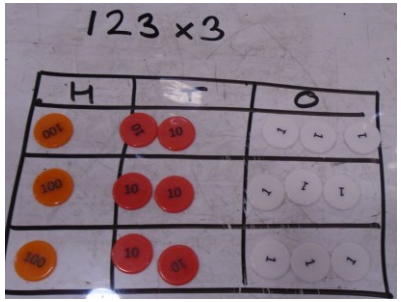
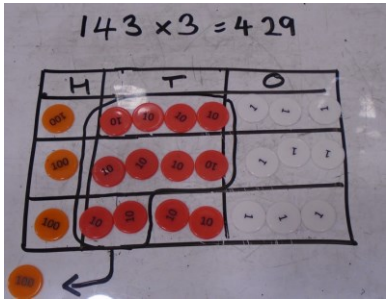
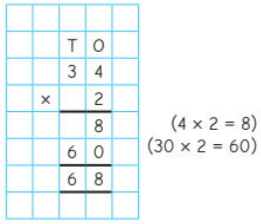
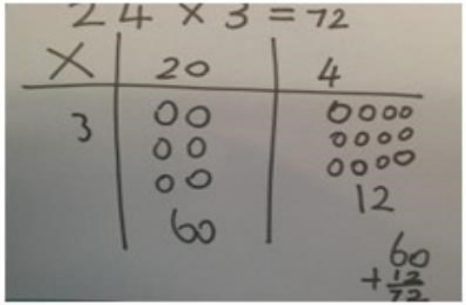
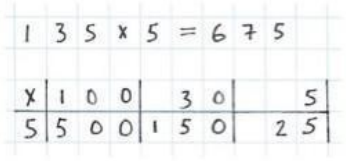
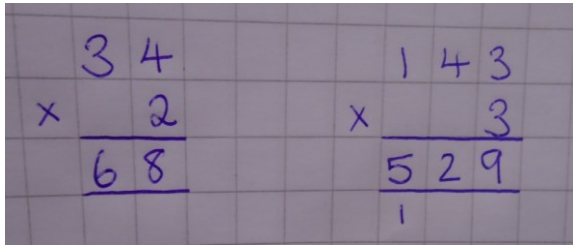
Multiplication- Year 3

Objective and Strategy	Concrete	Pictorial	Abstract
Multiplying two digit number by a one digit number *-no exchange* *-exchange* Grid method progressing to the formal method. Solving problems including missing number problems, integer scaling problems.	Show the link with arrays to first introduce the grid method.  Move on to using Base 10 to move towards a more compact m  Move on to place value counters to show how we are finding groups of a number.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Part-Whole Model (Pictorial):	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. TO x O Part-Whole Model (abstract) : 

	  (NOTE ABOVE – Show exchange of 10 ones as 1 Ten)	 Children can draw their place value grids and tens counters to show pictorial representation. Use brackets within expanded method – See below • Dora uses place value counters alongside the written multiplication to work out 34 × 2 Tens / Ones (4 × 2 = 8) (30 × 2 = 60)	Children to add up each column to find the answer (see below). T O 3 4 × 2 8 6 0 6 8 (4 × 2 = 8) (30 × 2 = 60)
Vocabulary	**Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up**		

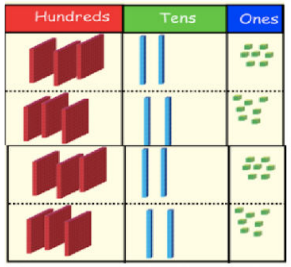
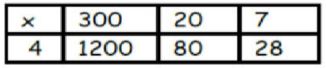
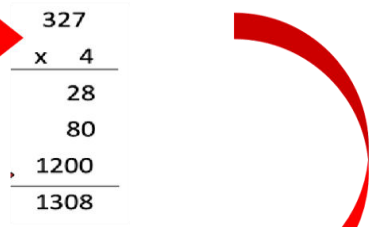
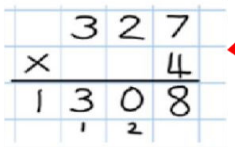
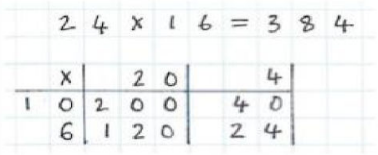
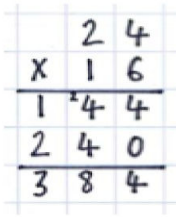
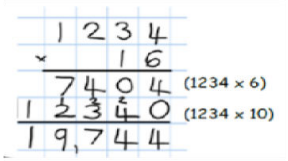
Multiplication- Year 4

Objective and Strategy	Concrete	Pictorial	Abstract

Multiply two digit and three-digit numbers by a one-digit number using formal written layout *-no exchange* *-exchange*	  	• Dora uses place value counters alongside the written multiplication to work out 34 × 2   (4 × 2 = 8) (30 × 2 = 60) Show multiplication with brackets including HTO x O. Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	HTO x O  Children to add up each column to find the answer. Multiplication - contracted  Vocab – Exchange, re-grouping

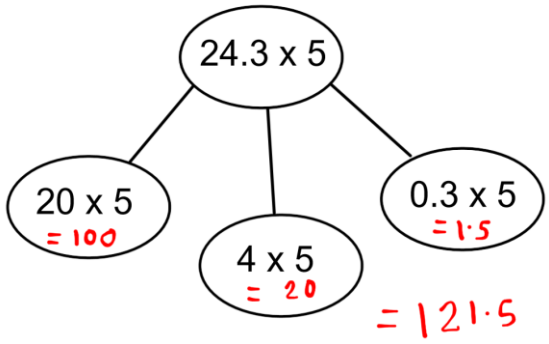
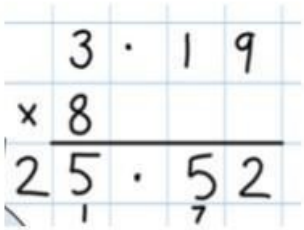
Multiplication - Year 5 (Begin teaching Year 6 strategies from Spring term)

Multiplication Year 5

Objective and Strategy	Concrete	Pictorial	Abstract
Multiply numbers up to 4-digits by a one-digit number using the format written method, including long multiplication for 2-digit numbers Column multiplication for 3 and 4 digits x 1 digit	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside		  This may lead to a compact method.
Column multiplication (long multiplication)	Manipulatives may still be used with the corresponding long multiplication modelled alongside **Model** | **Calculation**	Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 	 24 x 6 on the first row. (6 x 4 = 24, carrying the 2 for the 20, then 6 x 2) 24 x 10 on the second row. Show multiplying by 10 by putting zero in the units first. 
Vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed		

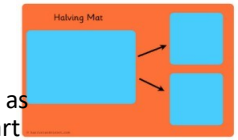
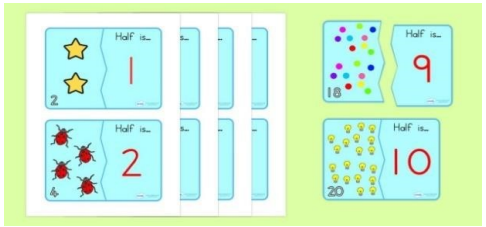
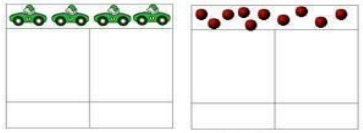
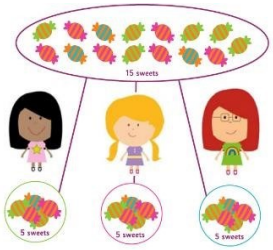
Vocabulary - double, near double, half, halve how many have gone? equals, Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed

Multiplication- Year 6

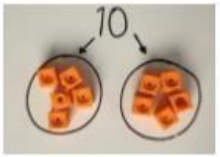
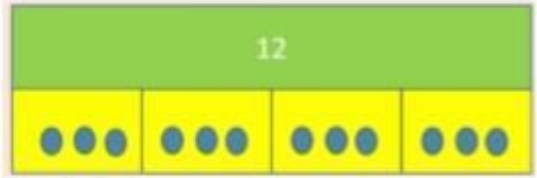
Objective and Strategy	Concrete	Pictorial	Abstract
Multiply decimal up to 2 decimal place by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 
Vocabulary	**Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed**		

Division

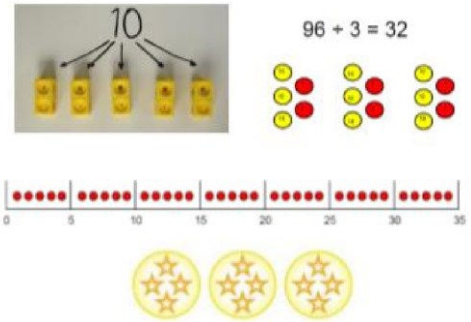
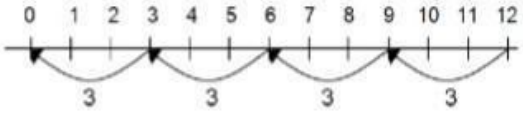
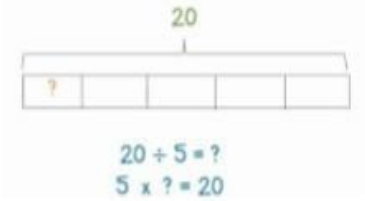
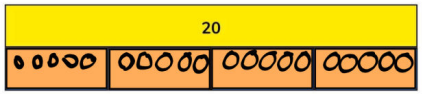
Division- EYFS			
Objectives	**Concrete**	**Pictorial**	**Abstract**

Solve problems including halving and sharing. Halving a whole, halving a quantity of objects. Sharing a quantity of objects.	   Children have the opportunity to physically cut objects, food or shapes in half. Counting and other  Use visual supports such as halving mats and part part whole, with the physical objects and resources that can be manipulated.    Counting and other maths resources for children to explore sharing between 3 or more. Maths resources for children to share into two equal groups.	 Pictures and icons that encourage children to see concept of halving in relation to subitising, addition and subtraction knowledge. i.e. Knowing 4 is made of 2 groups of 2, so half of 4 is 2.  Bar model with pictures or icons to support understanding of finding 2 equal parts of a number, to further understand how two halves make a whole.  Pictures for children to create and visualise 3 or more equal groups.	

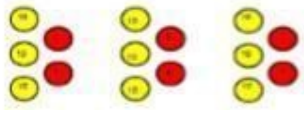
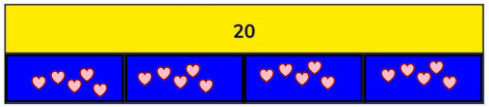
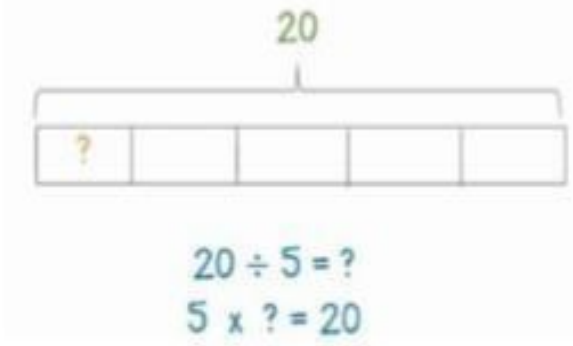
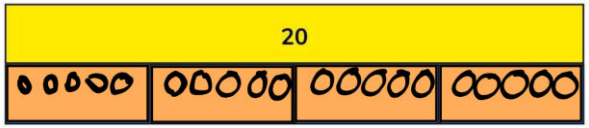
Division- Year 1

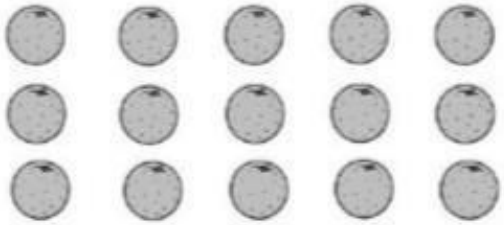
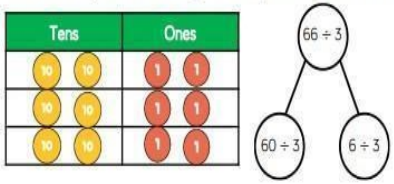
Objective and Strategy	Concrete	Pictorial	Abstract
Division as sharing (sharing objects into groups)	   I have 10 cubes, can you shar equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	
Vocabulary	**share, share equally, one each, two each…, group, groups of, lots of, array**		

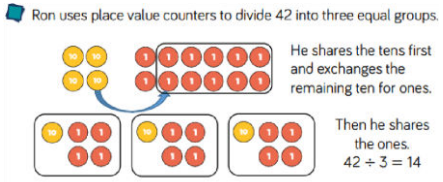
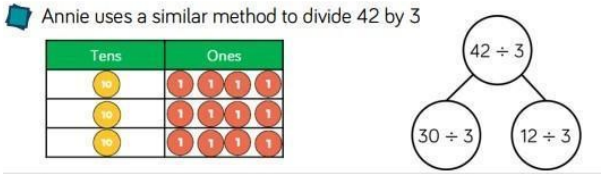
Division- Year 2

Objective and Strategy	Concrete	Pictorial	Abstract
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  Bar Model Using counters / cubes: **e.g. 20 ÷ 5 = 4** 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  Bar Model using annotations: **e.g. 20 ÷ 5 = 4** 	**Ensure that Concrete and Pictorial are secure before representing as Abstract.** **Share 9 buns between three people.** **9 ÷ 3 = 3** **28 ÷ 7 = 4** **Divide 28 into 7 groups. How many are in each group?** Bar Model with numbers / digits recorded: **e.g. 20 ÷ 5 = 4** 
Vocabulary	**share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over**		

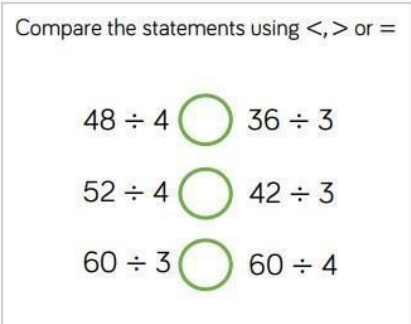
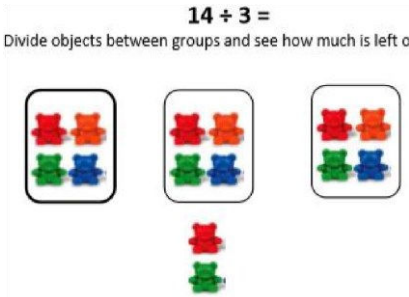
Division- Year 3

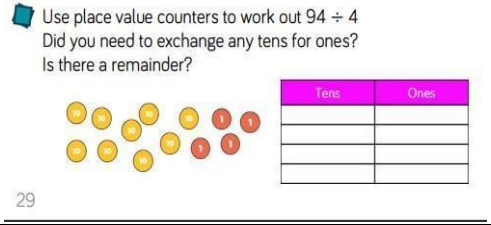
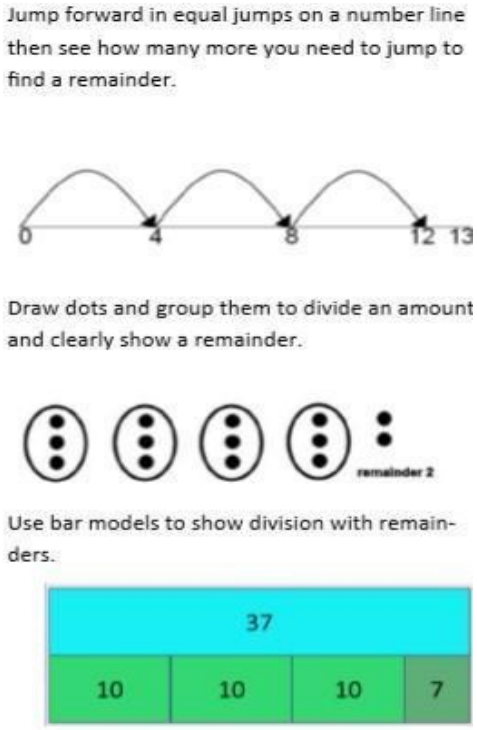
Objective and Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32  Bar Model Using counters / cubes: **e.g. 20 ÷ 5 = 4** 	Continue to use bar modelling to aid solving division problems.  Bar Model using annotations: **e.g. 20 ÷ 5 = 4** 	**Ensure that Concrete and Pictorial are secure before representing as Abstract.** How many groups of 6 in 24? 24 ÷ 6 = 4 Bar Model with numbers / digits recorded: **e.g. 20 ÷ 5 = 4** 

Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentenc-es that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4
Divide 2 digit numbers by a 1 digit number by partitioning into tens and ones using a pv grid	' Eva uses a place value grid and part-whole model to solve 6 	See part- whole model	

<table>
<tr>
<td>Divide numbers that involve exchanging between the tens and ones. The answers do not have remainders.</td>
<td> 
</td>
<td> 

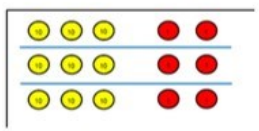
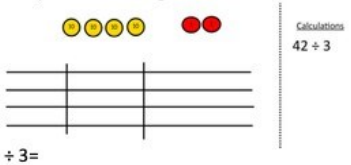
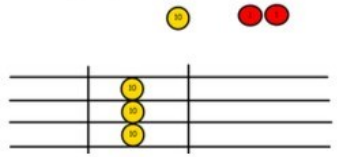
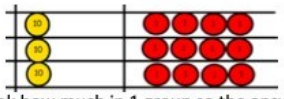
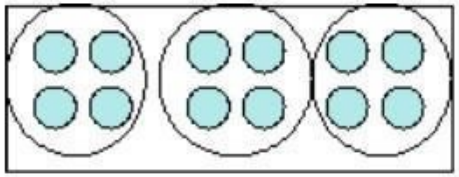
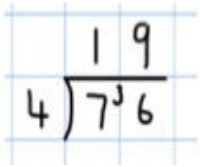
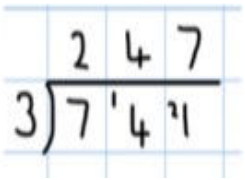
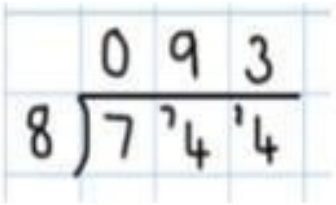
Children may use pictorial representation for the pv counters, alongside the part-whole model

Children use their times-tables to partition the number into multiples of the divisor.</td>
<td>96 ÷ 8
96 ÷ 4
96 ÷ 3
96 ÷ 6
 
</td>
</tr>
<tr>
<td>Division with a remainder</td>
<td> 

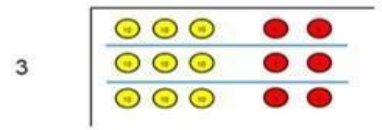
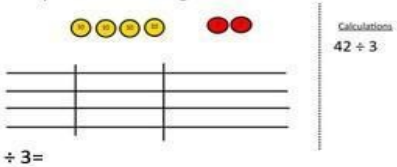
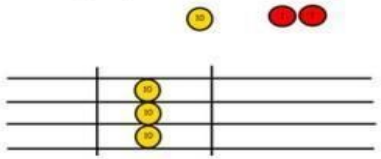
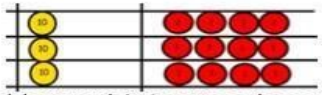
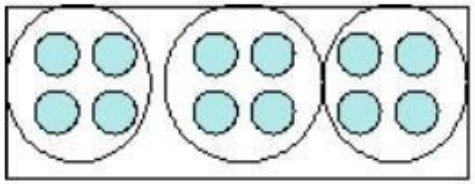
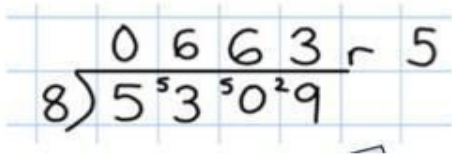
Moving on to:
 
</td>
<td> 
</td>
<td>Complete written divisions and show the remainder using ‘r’. e.g.

29 ÷ 8 = 3 r 5</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product</td>
</tr>
</table>

Division- Year 4

Objective and Strategy	Concrete	Pictorial	Abstract
Divide up to 3 digit numbers by 1 digit. Short (Bus Stop) Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. Bar Model	Begin with divisions that divide equally with no remainder – in tandem with pictorial representation, e.g. Place Value Grid (See left)   Children should be aware that a 0 is used to keep place value, if the number is not divisible. 
Vocabulary	**share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive**		

Division- Year 5 (Begin teaching Year 6 strategies from Spring term)

Objective and Strategy	Concrete	Pictorial	Abstract
Divide at least 4 digit numbers by 1 digit. Interpret remainders appropriately for the context Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide with and without remainder **Divide by 1 digit = use compact Bus Stop**  **Divide by 2 digits = Use long / extended division** 432 ÷ 15 becomes 2 8 r12 15 | 4 3 2 3 0 0 1 3 2 1 2 0 1 2 *Answer:* 28 remainder 12 Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 35 | 5 1 1 . 0
Vocabulary	**share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.**		

Division-Year 6 (Begin teaching Year 6 strategies from Spring term)

Objective and Strategy	Abstract
Long Division	h t o 0 4 1 R1 4) 1 6 5 4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160). 4 goes into 16 four times. 4 goes into 5 once, leaving a remainder of 1. th h t o 0 4 0 0 R7 8) 3 2 0 7 8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200). 8 goes into 32 four times (3,200 ÷ 8 = 400) 8 goes into 0 zero times (tens). 8 goes into 7 zero times, and leaves a remainder of 7. h t o 0 6 1 4) 2 4 7 − 4 3 When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3. Check: 4 × 61 + 3 = 247 th h t o 0 4 0 2 4) 1 6 0 9 − 8 1 When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1. Check: 4 × 402 + 1 = 1,609

Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.

Minimal Resources required to support the CPA approach (depending on year group):

- 10 frames (including egg boxes)
- Straws/pipe cleaners
- Bead strings (to 20 and 100)
- Rekenrek frames
- Base 10/Dienes (including magnetic to model on flip chart)
- Place value grids
- Double-sided counters
- Part-part whole templates
- Place value counters (KS2)
- Multi-link cubes